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Observer design of discrete-time impulsive switched nonlinear systems with time-varying delays

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ABSTRACT

This paper investigates the problem of observer design for discrete-time impulsive switched nonlinear systems with time-varying delays. Firstly, based on the average dwell time approach and the Lyapunov–Krasovskii functional technique, a delay-dependent exponential stability criterion for the discrete-time impulsive switched nonlinear systems with time-varying delays is derived in terms of a set of linear matrix inequalities (LMIs). Then, sufficient conditions for the existence of an observer that guarantees the exponential stability of the corresponding error system are proposed. Finally, two numerical examples are given to illustrate the effectiveness of the proposed method.

1. Introduction

Switched systems are a class of important hybrid systems consisting of subsystems and a switching law which defines a specific subsystem being activated during a certain interval of time [1]. Due to the theoretical development as well as practical applications, analysis and synthesis of switched systems have recently gained considerable attention, and a large number of results in this field have appeared (see, e.g., [2–6]). However, in the real world, some switched systems exhibit impulsive dynamical behavior due to sudden changes in the states of these systems at certain instants of switching, this kind of systems are usually called impulsive switched systems [7]. During the last decade, impulsive switched systems have received considerable attention. Some results on stability and stabilization of such systems have been achieved (see, e.g., [8–10]). For instance, stability analysis of systems with impulse effects was investigated in [8]. Analysis and design of impulsive control systems were considered in [10].

In many practical systems, there inevitably exists time delay which plays a crucial role in destroying the system performance. Although, in recent years, many stability conditions of time-delay systems have been obtained (see, e.g., [11–16]), it is worth mentioning that till now, there is only little effort putting on impulsive switched systems with time delays [17–27]. For example, some stability criteria for impulsive switched systems with constant time delays were developed in [17–26]. A delay-dependent stability criterion for a class of impulsive switched discrete systems with time-varying delays was established by using the Lyapunov–Krasovskii functional technique in [27].

On the other hand, it is necessary to design state observers for the systems due to the fact that the states of the systems are not all measurable in practice. Some significant results on this topic have been obtained in [28–33]. For example, the asymptotic stability property of the proposed switching observer was discussed and an LMI-based algorithm was given for a class of impulsive switched systems in [33]. However, to the best of our knowledge, the problem of observer design for discrete-time impulsive switched systems has not been fully investigated, especially for impulsive switched nonlinear systems with time-varying delays, which motivates us for this study.

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In this paper, we are interested in investigating the observer design problem for impulsive switched nonlinear systems with time-varying delays. The main contributions of this paper are as follows: (1) based on the dwell time approach, we establish a delay-dependent exponential stability criterion of the underlying system, which is different from those proposed in [33]; and (2) an observer design scheme for the underlying system is proposed.

The remainder of the paper is organized as follows. In Section 2, the problem formulation and some necessary lemmas are presented. In Section 3, the main results are obtained. Section 4 gives two numerical examples to illustrate the effectiveness of the proposed approach. Finally, concluding remarks are given in Section 5.

Notations. Throughout this paper, the superscript "T" denotes the transpose, and the notation $X \geqslant Y (X > Y)$ means that matrix $X - Y$ is positive semi-definite (positive definite, respectively). $\| \cdot \|$ denotes the Euclidean norm. I represents the identity matrix. $diag\{b_i\}$ denotes a diagonal matrix with the diagonal elements $b_i, i = 1, 2, \ldots, n$. X^{-1} denotes the inverse of X. The asterisk $*$ in a matrix is used to denote the term that is induced by symmetry. Z_0^+ denotes the set of all nonnegative integers. The set of all positive integers is represented by Z^+.

2. Problem formulation and preliminaries

Consider the following discrete-time impulsive switched nonlinear systems with time-varying delays:

$$x(k+1) = A_{\sigma(k)}x(k) + A_{d\sigma(k)}x(k-d(k)) + B_{\sigma(k)}u(k) + D_{f\sigma(k)}f_{\sigma(k)}(x(k)), \quad k \neq k_b - 1, \quad b \in Z^+, \tag{1a}$$

$$x(k+1) = E_{\sigma(k+1)\sigma(k)}x(k), \qquad k = k_b - 1, \quad b \in Z^+, \tag{1b}$$

$$y(k) = C_{\sigma(k)}x(k), \tag{1c}$$

$$x(k_0 + \theta) = \phi(\theta), \quad \theta \in [-d_2, 0], \tag{1d}$$

where $x(k) \in R^n$ is the state vector, $u(k) \in R^m$ is the control input, $y(k) \in R^p$ is the output, $\phi(\theta)$ is a discrete vector-valued initial function defined on the interval $[-d_2, 0]$. $d(k)$ is the discrete time-varying delay which is assumed to satisfy $d_1 \leqslant d(k) \leqslant d_2$. d_1 and d_2 are constant positive scalars representing the minimum and maximum delays, respectively. k_0 is the initial time. The function $\sigma(k) : Z_0^+ \to \underline{N} := \{1, \ldots, N\}$ is the switching signal with N being the number of subsystems. For each $i \in \underline{N}$, $f_i(x(k)) \in R^n$ is a known nonlinear function. A_i, A_{di}, B_i, D_{fi}, E_{ji} and C_i, $i, j \in \underline{N}, i \neq j$, are known real constant matrices with appropriate dimensions.

Remark 1. It should be noted that Eq. (1b) has been introduced in the literature [25–26,33] to describe the impulsive dynamical behavior of some practical switched systems due to sudden changes in the state of the system at certain instants of switching.

Remark 2. In the paper, it is assumed that the switching occurs at the instant k_b. The switching sequence of the system can be described as

$$\Sigma = \{(k_0, \sigma(k_0)), (k_1, \sigma(k_1)), (k_2, \sigma(k_2)), \cdots, (k_b, \sigma(k_b)), \cdots\}, \tag{2}$$

where k_b denotes the b-th switching instant.

We construct the following discrete-time impulsive switched systems to estimate the state of system (1):

$$\hat{x}(k+1) = A_{\sigma(k)}\hat{x}(k) + A_{d\sigma(k)}\hat{x}(k-d(k)) + B_{\sigma(k)}u(k) + L_{\sigma(k)}(y(k) - \hat{y}(k)) + D_{f\sigma(k)}\hat{f}_{\sigma(k)}(\hat{x}(k)), \quad k \neq k_b - 1, \quad b \in Z^+, \tag{3a}$$

$$\hat{x}(k+1) = H_{\sigma(k+1)\sigma(k)}\hat{x}(k), \quad k = k_b - 1, \quad b \in Z^+, \tag{3b}$$

$$\hat{y}(k) = C_{\sigma(k)}\hat{x}(k), \tag{3c}$$

$$\hat{x}(k_0 + \theta) = 0, \quad \theta \in [-d_2, 0], \tag{3d}$$

where $\hat{x}(k) \in R^n$ is the estimated state vector of $x(k)$, $\hat{y}(k) \in R^p$ is the observer output vector. For each $i \in \underline{N}$, $\hat{f}_i(\hat{x}(k)) \in R^n$ is a known nonlinear function. $L_i \in R^{n \times p}$ and $H_{ij} \in R^{n \times n}$, $\forall i, j \in \underline{N}, i \neq j$, are the matrices to be determined.

Let $\tilde{x}(k) = x(k) - \hat{x}(k)$ be the estimated error, then we can obtain the following error system:

$$\tilde{x}(k+1) = (A_{\sigma(k)} - L_{\sigma(k)}C_{\sigma(k)})\tilde{x}(k) + A_{d\sigma(k)}\tilde{x}(k-d(k)) + D_{f\sigma(k)}(f_{\sigma(k)}(x(k)) - \hat{f}_{\sigma(k)}(\hat{x}(k))), \quad k \neq k_b - 1, \quad b \in Z^+, \tag{4a}$$

$$\tilde{x}(k+1) = E_{\sigma(k+1)\sigma(k)}x(k) - H_{\sigma(k+1)\sigma(k)}\hat{x}(k), \quad k = k_b - 1, \quad b \in Z^+, \tag{4b}$$

$$\tilde{x}(k_0 + \theta) = \phi(\theta), \quad \theta \in [-d_2, 0] \tag{4c}$$

Before ending this section, we introduce the following definitions and lemmas.