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Non-fragile observer-based sliding mode control for Markovian jump systems with mixed mode-dependent time delays and input nonlinearity

Lijun Gao *, Dandan Wang, Yuqiang Wu

Department of Electrical Engineering and Automation, Qufu Normal University, Rizhao 276826, Shandong, People's Republic of China

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ABSTRACT

This paper is concerned with the state estimation and the sliding mode control (SMC) for a class of Markovian jump systems with mixed mode-dependent time delays and input nonlinearity. A non-fragile observer is designed to estimate the system states, and an observer-based SMC is synthesized to ensure the reachability of the sliding surfaces in the state-estimation space. By constructing a novel mode-dependent Lyapunov–Krasovskii functional, some new criteria are established for the existence and the solvability of the desired observer-based sliding mode controller. Finally, numerical examples are presented to illustrate the effectiveness and less conservativeness of the proposed theoretical results.

1. Introduction

Markovian jump systems constitute one of the classes of hybrid systems that are subject to random abrupt changes in their structure [1]. These changes are usually caused by random failure or repairs of the components, changing in subsystems interconnections, sudden environmental changes, etc. The class of jump system was firstly introduced in the early 1960s by Krasovskii and Lidskii [2]. During the past decades, due to their potential applications in manufacturing systems and communications, jump systems have been applied to model various dynamical systems, such as manufacturing systems, aircraft control, target tracking, solar receiver control and power systems [3–6]. On the theoretical front, the problems in [7–15], concerned stability and stabilization, control and filtering, robust control and adaptive control were well investigated.

It is well known that time delays and uncertainties are frequently encountered in a variety of control systems, such as vehicle active suspension systems, electric power and network control systems, etc. Time delays occur unavoidably due to signal transmission, inevitable defects of control equipment and so on. Time delays, either constant or time varying, can degrade the performance of control systems designed without considering the delays and can even destabilize the systems. Hence, time-delay systems have attracted considerable attention and motivate a chain of researches [16–24]. Furthermore, as a combination of both discrete and distributed delays, the so-called mixed time-delays have gained much research attention and many relevant results have been reported, (see e.g., [25,26]) for more details. Mixed mode-dependent time delays are of practical significance since the signal may switch between different modes and also propagate in a distributed way during a certain time period with the presence of an amount of parallel pathway [27–29]. Because sliding mode control (SMC) has attractive features, such as fast response and good transient response, and it is also insensitive to variations in system parameters and external disturbances, SMC has been widely recognized as an effective tool to deal with time delay systems (see e.g., [30–33]). So far, literatures [34–36] concerned SMC for Markovian jump systems with time delays have

* Corresponding author.
E-mail address: gljwg1977@163.com (L. Gao).

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been investigated. It should be noted that the results mentioned above based on the condition of system state are observable; nevertheless, in fact, this assumption is not reasonable in real systems. This has motivated the development of SMC based on observers [37–40]. However, it should be noted that the considered time delay of [37–39] is time invariant, which limits the scope of applications of the proposed results. In [40], the delay is mixed and not mode-dependent, which will bring more conservativeness to the actual applications. It is worth mentioning that most of results are applicable to non-Markovian jump systems only, and the relevant results for Markovian jump systems with mixed mode-dependent delay have been very few. Such a situation gives us the initial motivation for establishing a unified framework in order to handle the mixed mode-dependent delay for Markovian jump systems by using observer-based SMC scheme.

This is a consensus that the effects of input nonlinearity (e.g., saturation and dead-zones) must be taken into account when analyzing and implementing a SMC scheme, (see e.g., [41,42]) for more details. In recently years, attention has been paid to input nonlinearity ([42–46]), but few works undertaken on SMC for time-delay systems about unknown state subjected to input nonlinearity, and the existence of the perturbations also add the complexity of SMC design. Liu et al. [47,48] studied non-fragile observer-based SMC of uncertain systems subjected to input nonlinearity, where the time delay is time invariant, and a delay-independent sufficient condition is proposed. In a general way, the delay-independent criteria cannot completely reflect the information on the length of delays and it is more conservative than delay-dependent ones. Although the above mentioned SMC is robust to uncertainties existing in the plant, the robustness with respect to uncertainties existing in the controller has not been considered. In [49–52], non-fragile controller is designed such that the controller is insensitive to some amount of error with respect to its gain. Unfortunately, until now, the non-fragile SMC control for Markovian systems have received little attention. Motivated by the above discussion, we study this paper. The objective of this paper is to investigate the SMC observer-based problem for Markovian jump systems with mixed mode-dependent delays and input nonlinearity.

The main contributions of this paper can be listed as follows: (1) a new Lyapunov–Krasovskii functional is proposed; (2) SMC observer-based problem for Markovian jump systems with mixed mode-dependent delays and input nonlinearity is studied; (3) the result developed in this paper is less conservativeness about delay-dependent than those in the literature. We derive a sufficient condition to guarantee that the addressed Markovian jump system is stochastically stable. Finally, numerical simulations are presented to further demonstrate the effectiveness and the less conservativeness of the proposed approach.

Notations. The notations in this paper are fairly standard. For real symmetric matrices X and Y, the notation $X \geqslant Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite); A^T represents the transpose of matrix A. Let $\mathbb{R}$ denote the set of real numbers; $\mathbb{R}^n$ stands for the n-dimensional Euclidean space; $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices. The notation $\mathrm{diag}(\cdot)$ is a diagonal matrix. In symmetric block matrices, we use an asterisk $(*)$ to represent a term that is induced by symmetry. $\|\cdot\|$ and $\|\cdot\|_1$ denote the Euclidean norm and 1-norm of a vector or its induced matrix norm respectively, and I refers to an identity matrix with appropriate dimensions. If $a \in R^n$, we have $\|a\| < \|a\|_1$. $\|\cdot\|_2$ stands for the usual $L_2[0,\infty]$ norm. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. Problem formulation

Given a probability space (Ω, F, P) with Ω is the sample space, F the σ algebra of events, and P the probability measure defined on F. The Markov process $\{r_t, t \geqslant 0\}$ represents the switching between the different modes taking values in a finite state space $S = \{1, 2, \ldots, N\}$ with generator $\pi = (\pi_{ij})_{N\times N}$ given by

$$\Pr\{r_{t+\Delta} = j | r_t = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & \text{if } i \neq j, \\ 1 + \pi_{ii}\Delta + o(\Delta), & \text{if } i = j, \end{cases}$$

where $\Delta > 0, \lim_{\Delta\to 0} o(\Delta)/\Delta = 0$ and $\pi_{ij} \geqslant 0$, for $i \neq j$, is the transition rate from mode i to j and $\pi_{ii} = -\sum_{j\neq i}\pi_{ij}$.

Consider a class of Markovian jump systems with mixed mode-dependent time-varying delays and input nonlinearity described as

$$\begin{aligned} \dot{x}(t) = {} & (A(r_t) + \Delta A(r_t,t))x(t) + (A_1(r_t) + \Delta A_1(r_t,t))x(t - \tau_{1,r_t}(t)) + A_2(r_t) \\ & \int_{t-\tau_{2,r_t}(t)}^{t} f(x(\theta))d\theta + B(r_t)(\phi(u,r_t) + f_1(t,x(t),r_t)) + g_1(r_t)f_2(t,x(t)) + g_2(r_t)f_3(t,x(t-\tau_{1,r_t}(t))), \end{aligned} \tag{1}$$

$$y(t) = C(r_t)x(t), \tag{2}$$

$$x(t) = \varphi(t), \quad t \in [-\tau, 0], \tag{3}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input, $y(t) \in R^p$ is the measured output, $\varphi(t)$ is a continuous vector-valued initial function of $t \in [-\tau, 0]$, and $\phi(u, r_t)$ is continuous function vector. $A(r_t), A_1(r_t), A_2(r_t), B(r_t)$, and $C(r_t)$ are known mode dependent constant matrices with appropriate dimensions. In general, it is assumed that the pair $(A(r_t), B(r_t))$ is controllable and the input matrix $B(r_t)$ has full column rank. $\Delta A(r_t,t)$ and $\Delta A_1(r_t,t)$ are unknown time-varying matrices representing system parameter uncertainties, and that satisfy

$$[\,\Delta A(r_t,t) \quad \Delta A_1(r_t,t)\,] = M(r_t)F(r_t,t)[\,N_1(r_t) \quad N_2(r_t)\,], \tag{4}$$