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Asymptotic stability of cellular neural networks with multiple proportional delays ☆

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ABSTRACT

Proportional delay is a kind of unbounded time-varying delay, which is different from unbounded distributed delays. In this paper, asymptotic stability of the equilibrium point of cellular neural networks with multiple proportional delays is presented. Sufficient conditions for delay-dependent global asymptotic stability and delay-independent uniform asymptotic stability of the system are obtained by employing matrix theory and constructing Lyapunov functional. Two examples are given to illustrate the effectiveness of the obtained results and less conservative than previously existing results.

1. Introduction

Cellular neural networks (CNNs) [1] have been widely investigated owing to their widespread applications in image processing, pattern recognition, optimization and associative memories. These practical application of CNNs depends on the existence and stability of the equilibrium point of the CNNs. Moreover, time delay is inevitable due to the finite switching speed of information processing and the inherent communication time of neurons, and its existence may cause the instability of the system. Therefore, a number of stability criteria of CNNs with delays have been proposed [2–20]. At present, stability results of CNNs with delays studied in [4,6,11,13,16,18] are mainly based on such approaches as M-matrix, algebraic inequalities, and so on. As pointed out in [14], the characteristic of those results, which means to take absolute value operation on the interconnection matrix, leads to the ignorance of neuron's inhibitory and excitatory effects on neural networks. In recent years, linear matrix inequality (LMI) technique has been used to deal with the stability problem for neural networks [2,3,5,7–10,12,14,15,17,19,20]. The feature of LMI-based results is that it can consider the neuron's inhibitory and excitatory effects on neural networks. However, few stability results have been obtained for CNNs with proportional delays on basis of LMI, which is important, as did for neural networks model studied in [32,33]. The exponential stability of CNNs with multi-pantograph delays was studied by nonlinear measure in [32]. In [33], by employing matrix theory and Lyapunov functional, global exponential stability of a class of CNNs with multi-proportional delays was investigated and delay-dependent sufficient conditions were obtained. The proportional delay system as an important mathematical model often rises in some fields such as physics, biology systems and control theory and it has attracted many scholars' interest [25–31].

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As a neural network usually has a spatial nature for the presence of an amount of parallel pathways of a variety of axon sizes and lengths, it is desired to model by introducing continuously proportional delay over a certain duration of time. The proportional delay function is that $\tau(t) = (1-q)t \to +\infty$ as $q \neq 1,\ t \to +\infty\ (0 < q \leqslant 1)$, so it is time-varying, unbounded, and monotonically increasing. Therefore, the network's running time may be controlled according to the network allowed delays. At the present time, besides the distributed delays, the delay function $\tau(t)$ is usually required to be bounded in the stability discussion of neural networks with delays, such as, these results in [9,12,19–24], are required that the delay function $\tau(t)$ satisfies $0 \leqslant \tau(t) \leqslant \tau$ and other conditions. It is seldom that the delay function $\tau(t)$ is unbounded, i.e. $\tau(t) \to +\infty (t \to +\infty)$. Compared with the distributed delay [6–8,10,13], whose delay kernel functions satisfy some conditions such that the distributed delay is easier to be handled [8,10,13] in the use inequality, but the proportional delay is not easy to be controlled. Thus, it is important to study stability of neural networks with proportional delays in theory and practice.

Motivated by the discussion above, the asymptotic stability of CNNs with multiple proportional delays is further discussed in this paper, inspired by Liao et al. [4], Zhang and Wang [14] and Zhang and Zhou [32]. This paper is organized as follows. Model description and preliminaries are given in Section 2. Two delay-dependent and delay-independent sufficient conditions are given in Section 3 to ascertain the asymptotic stability of the CNNs with multiple proportional delays, which are easy to be verified. Numerical examples and their simulation are presented in Section 4 to illustrate the effectiveness and less conservative of obtained results. Finally, conclusions are given in Section 5.

2. Model description and preliminaries

Consider the following CNNs with multiple proportional delays

$$\begin{cases} \dot{x}_i(t) = -d_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(q_{ij}t)) + I_i, \\ x_i(s) = x_{i0}, \quad s \in [q, 1],\ i = 1, 2 \ldots, n \end{cases} \tag{2.1}$$

for $t \geqslant 1$, where n is the number of neurons; $d_i > 0$ is a constant; $x_i(t)$ is the state variable; a_{ij} and b_{ij} are constants which denote the strengths of connectivity between the cells j and i at time t and $q_{ij}t$, respectively; $f_i(\cdot)$ denotes a nonlinear activation function; I_i denotes the constant external inputs; $q_{ij},\ i,j = 1,2,\ldots,n$ are proportional delay factors and satisfy $0 < q_{ij} \leqslant 1$, and $q_{ij}t = t - (1-q_{ij})t$, in which $\tau_{ij}(t) = (1-q_{ij})t$ is the transmission delay function, and $(1-q_{ij})t \to +\infty$ as $q_{ij} \neq 1,\ t \to +\infty, q = \min_{1 \leqslant i,j \leqslant n}\{q_{ij}\}$; x_{i0} denotes constant initial value of $x_i(s)$ at $s \in [q,1]$. Assume that $f_j(\cdot),\ j = 1,2,\ldots,n$ are bounded, monotonically nondecreasing and satisfying

$$0 \leqslant \frac{f_i(u) - f_i(v)}{u - v} \leqslant l_i, f_i(0) = 0, \quad u \neq v;\ f_i(u) \neq 0,\ u \neq 0,\ u,\ v \in \mathbb{R}. \tag{2.2}$$

Remark 2.1. In (2.1), if $q_{ij} = 1,\ i,j = 1,2,\ldots,n$, then system (2.1) is a class of standard cellular neural networks model.

Throughout this paper, the notation $A > 0$ means that the matrix A is symmetric positive definite. The notation A^T and A^{-1} denote the transpose and the inverse of a square matrix A, respectively. $\mathbb{R}^n$ denotes the n-dimension Euclidean Space, $\mathbb{R}^+ = [0, +\infty),\ \|x\|$ denotes the Euclidean norm in $\mathbb{R}^n,\ x \in \mathbb{R}^n$. $C([-\tau, 0], \mathbb{R})$ represents a set of all continuous functions from $[-\tau, 0]$ to $\mathbb{R}$.

It is known that there always exists an equilibrium point for system (2.1). Let $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ be an equilibrium point of (2.1). By shifting x^* to the origin, then (2.1) are converted into the following form

$$\begin{cases} \dot{z}_i(t) = -d_i z_i(t) + \sum_{j=1}^{n} a_{ij} g_j(z_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(z_j(q_{ij}t)), \\ z_i(s) = z_{i0}, \quad s \in [q, 1],\ i = 1, 2, \ldots, n, \end{cases} \tag{2.3}$$

where $z_i(\cdot) = x_i(\cdot) - x_i^*$, and $g_j(z_j(\cdot)) = f_j(z_j(\cdot) + x_j^*) - f_j(x_j^*)$.

Note that since each function $f_i(\cdot)$ satisfies the condition (2.2), hence, each g_i satisfies

$$g_i^2(z_i(\cdot)) \leqslant l_i z_i(\cdot) g_i(z_i(\cdot)), \quad g_i(0) = 0,\ i = 1, 2, \ldots, n. \tag{2.4}$$

Let $y_i(t) = z_i(e^t)$. It is easy to prove that system (2.3) is equivalent to the following CNNs with constant delays and variable coefficients (see [33])

$$\begin{cases} \dot{y}_i(t) = e^t \left\{ -d_i y_i(t) + \sum_{j=1}^{n} a_{ij} g_j(y_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(y_j(t - \tau_{ij})) \right\}, \quad t \geqslant 0, \\ y_i(s) = \varphi_i(s), \quad s \in [-\tau, 0],\ i = 1, 2, \ldots, n, \end{cases} \tag{2.5}$$

where $\tau_{ij} = -\log q_{ij} \geq 0,\ \tau = \max_{1 \leqslant i,j \leqslant n}\{\tau_{ij}\}$ and $\varphi_i(s) = z_{i0} \in C([-\tau, 0], \mathbb{R})$.