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Fractional order integral equations of two independent variables

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ABSTRACT

In this paper, we present some results concerning the existence, the uniqueness and the attractivity of solutions for some functional integral equations of Riemann–Liouville fractional order, by using some fixed point theorems.

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1. Introduction

Fractional integral equations have recently been applied in various areas of engineering, science, finance, applied mathematics, and bio-engineering and others. However, many researchers remain unaware of this field. There has been a significant development in ordinary and partial fractional differential and integral equations in recent years; see the monographs of Baleanu et al. [4], Kilbas et al. [13], Miller and Ross [15], Lakshmikantham et al. [14], Podlubny [20], Samko et al. [22]. Recently some interesting results on the attractivity of the solutions of some classes of integral equations have been obtained by Abbas et al. [1,2], Banaś et al. [5–7], Darwish et al. [8], Dhage [9–11], Pachpatte [18,19] and the references therein.

In [17], Mureşan proved some results concerning the existence, uniqueness, data dependence and comparison theorems, by applying some results from Picard and weakly Picard operators' theory [21], for the following functional integral equation of the form

$$x(t) = \alpha + f\left(x, \int_0^{g(t)} x(s)ds, x(h(t))\right); t \in [0, T], \tag{1}$$

where $T > 0$, $f : [0,T] \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$, $g, h : [0,T] \to [0,T]$. In this paper we improve the above results for the following partial integral equation of Riemann–Liouville fractional order of the form

$$u(x,y) = \mu(x,y) + f(x,y,I_\theta^r u(x,y), u(x,y)); \quad (x,y) \in J := [0,a] \times [0,b], \tag{2}$$

where $\theta = (0,0)$, $r = (r_1, r_2)$, $r_1, r_2 \in (0,\infty)$, $\mu : J \to \mathbb{R}^n$, $f : J \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ are given continuous functions, I_θ^r is the left-sided mixed Riemann–Liouville integral of order r.

Next, we prove some results concerning the existence and the attractivity of solutions for the following partial Riemann–Liouville fractional order integral equation of the form

$$u(x,y) = \mu(x,y) + f(x,y,I_\theta^r u(x,y), u(x,y)); \quad (x,y) \in J' := \mathbb{R}_+ \times [0,b], \tag{3}$$

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where $b > 0$, $\mathbb{R}_+ = [0, \infty)$ and $\mu : J' \to \mathbb{R}^n$, $f : J' \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ are given continuous functions.

Our investigations are conducted in Banach spaces with an application of Banach's contraction principle and Schauder's fixed point theorem for the existence and uniqueness of solutions of Eq. (2). We use the Schauder fixed point theorem for the existence of solutions of Eq. (3), and we prove that all solutions are globally asymptotically stable. Also, we present some examples illustrating the applicability of the imposed conditions.

2. Preliminaries

In this section, we introduce notations, definitions, and preliminary facts which are used throughout this paper. By $C(J)$ we denote the Banach space of all continuous functions from J into $\mathbb{R}^n$ with the norm

$$\|w\|_\infty = \sup_{(x,y)\in J} \|w(x,y)\|,$$

where $\|.\|$ denotes a suitable complete norm on $\mathbb{R}^n$.

Let E be the space of functions $w \in C(J)$, which fulfill the following condition:

$$\exists M \geqslant 0 : \|w(x,y)\| \leqslant M e^{\lambda(x+y)}, \quad \text{for } (x,y) \in J, \tag{4}$$

where λ is a positive constant. In the space E we define the norm

$$\|w\|_E = \sup_{(x,y)\in J} \{\|w(x,y)\| e^{-\lambda(x+y)}\}.$$

According to [16], $(E, \|.\|_E)$ is a Banach space. The above definition of $\|.\|_E$ is variant of Bielecki's norm. We note that the condition (4) implies that

$$\|w\|_E \leqslant M. \tag{5}$$

$L^1(J)$ is the space of Lebesgue-integrable functions $w : J \to \mathbb{R}^n$ with the norm

$$\|w\|_{L^1} = \int_0^a \int_0^b \|w(x,y)\| dy\, dx.$$

By $BC := BC(\mathbb{R}_+ \times [0,b])$ we denote the Banach space of all bounded and continuous functions from $\mathbb{R}_+ \times [0,b]$ into $\mathbb{R}^n$ equipped with the standard norm

$$\|u\|_{BC} = \sup_{(x,y)\in \mathbb{R}_+ \times [0,b]} \|u(x,y)\|.$$

For $u_0 \in BC$ and $\eta \in (0,\infty)$, we denote by $B(u_0, \eta)$, the closed ball in BC centered at u_0 with radius η.

Definition 2.1 [23]. Let $r = (r_1, r_2) \in (0,\infty) \times (0,\infty)$, $\theta = (0,0)$ and $u \in L^1(J)$. The left-sided mixed Riemann–Liouville integral of order r of u is defined by

$$(I_\theta^r u)(x,y) = \frac{1}{\Gamma(r_1)\Gamma(r_2)} \int_0^x \int_0^y (x-s)^{r_1-1}(y-t)^{r_2-1} u(s,t)\, dt\, ds,$$

where $\Gamma(.)$ is the (Euler's) Gamma function defined by $\Gamma(\xi) = \int_0^\infty t^{\xi-1} e^{-t} dt;\ \xi > 0$.

In particular,

$$(I_\theta^\theta u)(x,y) = u(x,y),\ (I_\theta^\sigma u)(x,y) = \int_0^x \int_0^y u(s,t)\, dt\, ds; \quad \text{for almost all } (x,y) \in J,$$

where $\sigma = (1,1)$.

For instance, $I_\theta^r u$ exists for all $r_1, r_2 \in (0,\infty)$, when $u \in L^1(J)$. Note also that when $u \in C(J)$, then $(I_\theta^r u) \in C(J)$, moreover

$$(I_\theta^r u)(x,0) = (I_\theta^r u)(0,y) = 0; \quad x \in [0,a],\ y \in [0,b].$$

Example 2.2. Let $\lambda, \omega \in (-1,\infty)$ and $r = (r_1, r_2) \in (0,\infty) \times (0,\infty)$, then

$$I_\theta^r x^\lambda y^\omega = \frac{\Gamma(1+\lambda)\Gamma(1+\omega)}{\Gamma(1+\lambda+r_1)\Gamma(1+\omega+r_2)} x^{\lambda+r_1} y^{\omega+r_2}, \quad \text{for almost all } (x,y) \in J.$$

Let G be an operator from $\Omega \subset BC;\ \Omega \neq \emptyset$ into itself and consider the solutions of equation

$$(Gu)(x,y) = u(x,y). \tag{6}$$

Now we review the concept of attractivity of solutions for Eq. (6).

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