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Heat transfer and flow of a slag-type non-linear fluid: Effects of variable thermal conductivity

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ABSTRACT

In this paper, we study the effects of variable thermal conductivity on the flow and heat transfer in a slag-type non-linear fluid down a vertical wall. The constitutive relation for the heat flux vector is assumed to be the Fourier's law of conduction with a variable thermal conductivity which includes the second order effects of the volume fraction. We numerically solve the non-dimensional form of the governing equations to study the effects of various dimensionless numbers on the velocity, temperature and volume fraction. The results indicate that the thermal conductivity plays a major role in the temperature distribution. Also, for certain values of the dimensionless numbers, minor differences are observed in the velocity and volume fraction distribution compared with the case of constant thermal conductivity.

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1. Introduction

Since various engineering problems involve the transport of mass and energy through particulate and granular media, it is of great importance to understand the role of heat transfer in the processing of such materials. One of the main areas of interest in energy related processes, such as power plants, atomization, alternative fuels, etc., is the use of slurries, specifically coal–water or coal–oil slurries, as the primary fuel. Some studies indicate that the viscosity of coal–water mixtures depends not only on the volume fraction of solids, the mean size, and the size distribution of the coal particles, but also on the shear rate, since the slurry behaves as a non-linear fluid. At the same time, there are studies which indicate that preheating the fuel results in better performance, and as a result of such heating, the viscosity changes (Gupta and Massoudi [6]). A similar situation, i.e., efficient heating or cooling of a liquid occurs in the flow of a thin film along a solid surface which is kept at a constant temperature.

In the mechanics of non-linear materials, the stress tensor and the heat flux vector are among the most important constitutive relations. In recent years, there has been an increasing need for more accurate models to represent the conductive and radiative heat transfer processes, in manufacturing of various new materials such as fiber-reinforced composites, microfabrication technologies, nanoscale thermal transport especially in semiconductor industry, micro-time heat transfer processes such as short-pulse laser applications and high-speed electronics (Tien and Chen [28], Tzou [30,31], Cahill et al. [3]). The success of such models depends on the thermo-physical property data used in the numerical simulations. To develop an accurate heat transfer model in any type of coal combustion or gasification process, the heat transfer and to some extent the rheological properties of ash and slag,[1] especially in high-temperature environments need to be understood and

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[1] Many researchers have defined the phenomenon of "slagging" as the deposition of ash in the radiative section of a boiler.

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properly modeled. The viscosity of slag and the thermal conductivity of ash deposits are among two of the most important constitutive parameters that need to be studied (see Wang and Massoudi [33], Massoudi and Wang [17]). The accurate formulation or representations of the (transport) properties of coal (and biomass for co-firing cases) present a special challenge in modeling efforts in computational fluid dynamics (CFD) applications. Studies have indicated that slag viscosity must be within a certain range of temperatures for tapping and the membrane wall to be accessible. As the operating temperature decreases, the slag cools and solid crystals begin to form. In such cases the slag should be regarded as a non-Newtonian suspension, consisting of liquid silicate and crystals. A better understanding of the rheological properties of the slag, such as yield stress and shear-thinning, are critical in determining the optimum operating conditions. As Rezaei et al. [21] indicate, the thermal conductivity of slag depends on the porosity, chemical composition, temperature of the deposit, etc. They observed that the thermal conductivity of ash increases with increasing temperature, but decreases with increasing porosity. Noticeably, the thermal conductivity of slags was found to be higher than that of the particulate structure in the porosity range 0.2–0.8.

In this paper we generalize the work of Miao et al. [19] and study the heat transfer in the flow of a slag-like non-linear fluid along a vertical wall, while using a variable thermal conductivity expression in the heat flux vector. The governing equations are introduced in Section 2. Brief discussions on the constitutive relations are given in Section 3. Section 4 describes the geometry of the flow and non-dimensionalization of the governing equations. The effects of radiation are incorporated as a boundary condition. In Section 5, numerical results are presented and the effects of various dimensionless parameters on the velocity field, temperature and volume fraction are discussed.

2. Governing equations of motion and heat transfer

In the absence of any electro-magnetic effects, the governing equations of motion are the conservation of mass, linear momentum, and energy equations (Slattery [25]), which are given as:

2.1. Conservation of mass

$$\frac{\partial \rho}{\partial t} + div(\rho \mathbf{u}) = 0 \tag{1}$$

where ρ is the density of the fluid, $\partial/\partial t$ is the partial derivative with respect to time, *div* is the divergence operator, and $\mathbf{u}$ is the velocity vector. For an isochoric motion (incompressible material), the above equation reduces to $div\ \mathbf{u} = 0$.

2.2. Conservation of linear momentum

$$\rho \frac{d\mathbf{u}}{dt} = div\mathbf{T} + \rho \mathbf{b} \tag{2}$$

where $\mathbf{b}$ is the body force vector, $\mathbf{T}$ is the Cauchy stress tensor, which will be given by a constitutive equation, and d/dt is the total time derivative, given by $\frac{d(\cdot)}{dt} = \frac{\partial(\cdot)}{\partial t} + [grad(\cdot)]\mathbf{u}$. The balance of moment of momentum reveals that, in the absence of couple stresses, the stress tensor is symmetric.

2.3. Conservation of energy

$$\rho \frac{d\varepsilon}{dt} = \mathbf{T} : \mathbf{L} - div\mathbf{q} + \rho r \tag{3}$$

where ε is the specific internal energy, $\mathbf{L}$ is the velocity gradient, $\mathbf{q}$ is the heat flux vector, and r is the specific radiant energy. Thermodynamical considerations require the application of the second law of thermodynamics or the entropy inequality. The local form of the entropy inequality is given by (see Liu [12], p. 130):

$$\rho \frac{d\eta}{dt} + div\boldsymbol{\varphi} - \rho s \geqslant 0 \tag{4}$$

where $\eta(\mathbf{x}, t)$ is the specific entropy density, $\boldsymbol{\varphi}(\mathbf{x}, t)$ is the entropy flux, and s is the entropy supply density due to external sources and d/dt denotes the material time derivative. If it is assumed that $\varphi = \frac{\mathbf{q}}{\theta}$, and $s = \frac{r}{\theta}$, where θ is the absolute temperature, then Eq. (4) reduces to the Clausius–Duhem inequality:

$$\rho \frac{d\eta}{dt} + div\frac{\mathbf{q}}{\theta} - \rho \frac{r}{\theta} \geqslant 0 \tag{5}$$

Even though we do not consider the effects of the Clausius–Duhem inequality in our problem, for a complete thermo-mechanical study of this problem, the second law of thermodynamics must be considered (Liu [12]). To achieve "closure" for these equations, constitutive relations are needed for $\mathbf{T}$, r, and $\mathbf{q}$. In the next section we will discuss briefly the constitutive modeling issues.