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Optimal repair strategies for a two-unit deteriorating standby system

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ABSTRACT

In this paper, two optimal problems for a system composed of a main unit and a standby with common cause failure and critical human error are considered. The optimal repair rates for the optimal reliability and the optimal balance between system availability and repair cost are respectively discussed by constructing suitable objective functions and corresponding permitted sets with functional analysis method. Some numerical examples are presented at the end of the paper.

1. Introduction

Repairable system, as one of important systems in reliability theory, usually consists of four basic components, e.g. hardware, software, organizational structure and human. Reliability theory was originally developed to handle hardware failure of a system. However, the reasons of failure of modern systems are very complex. They could be caused by one or more of the components mentioned above. In the selection of a proper repair policy for a system, one must consider what, how and how often to inspect and maintain. The selection logic is to reach an efficient operation throughout a system's service life in compliance with given requirements and acceptance criteria which are typically related to allowable financial budget since the repair policy is usually different for various types and degree of failure and the cost and benefit of different repair policies vary. In other words, the repair policy needs to be applied so that a balance is achieved between the expected benefits and the corresponding finical cost.

From the point of view of production company, on the other hand, the primary objective of any task and operation on its system and components is to maximize production profit and minimize all losses. To this aim, maintenance must ensure that the system and components reliability characteristics be kept consistent with the requirements of the planned production and regulatory directives, at a minimum resource cost. The goal of effective maintenance planning is then minimizing unplanned downtime. In practice, taking into consideration of revenue, the maintenance philosophy of a production plant basically boils down to performing the optimal maintenance plan that is consistent with the optimal of production and plant availability, while not compromising safety and the associated regulatory requirements.

Therefore, for the importance in both practice and theory, optimal design of systems is increasingly emphasized. The criteria to be optimized are reliability, cost, weight or volume. System availability can be increased by either using components of lower failure rates, using better repair facilities or using standby redundant components. For systematic information, please refer to [1], in which Kou and Zou provided a complete picture of reliability evaluation and optimal system design for many well-studied system structures including parallel, series, standby, k-out-of-n, multi-state, and

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general system models. Muth [2] presented an optimal policy to minimize the cost per unit time for repair and replacement. Phelps [3] studied the optimal policy for minimal repair to make the total expected discounted cost minimal. Jayabalan and Chaudhuri [4] presented an algorithm which effectively curtails, and further determines the number of maintenance interventions before each replacement in order to minimize the total cost over a finite time horizon. Chiang and Yuan [5] proposed a state-dependent maintenance policy for a multi-state continuous-time Markovian deteriorating system subject to aging and fatal shocks to make the expected long-run cost rate minimized. Zhang and Wang [6–8] studied different repairable systems by using geometric process and supplementary variable technique, and developed unitary and mixed optimal replacement policies such that the long-run average cost per unit time of the systems is minimized.

In this paper, two optimal problems are considered based on the model proposed by Mendus et al. [9]. One is to find the optimal repair rate by which the system's steady-state reliability can reach the predefined value with convex analysis method. The other is to obtain the optimal repair rate by which a balance can be achieved between the maximal system's steady-state availability and the minimal repair cost with functional analysis method. Mendus et al. [9] provided a two-unit deteriorating standby system model with partial differential equations and studied the steady-state availability and mean-time to failure of the system by method of Laplace transform. Yuan and Xu [10] later discussed the existence and uniqueness of the solution of this system by using C_0-semigroup theory which is an essential basis for traditional Laplace transform technique. This paper is a further work of Yuan and Xu [10].

Rest of the paper is organized as follows. In Section 2, we introduce the system model and recall some results concerned. In Section 3.1, one optimal problem on the maximum system reliability is discussed by choosing a suitable objective set. In Section 3.2, the other optimal problem on the balance between the system availability and its repair cost is analyzed by constructing an objective function. Numerical examples are presented in Section 4, and Section 5 concludes the paper.

2. System model and basic results

Let us recall the model under consideration [9]. The system consists of a operating unit with failure rate λ and a standby unit. Initially, the system is in working order and this state is denoted by state 0. The standby unit can deteriorate in its standby mode in which it may fail with a failure rate λ_0. The failure of the standby unit and operating unit in state 0 respectively brings the system to state 1 and state 2 where the standby unit starts to work with an increased failure rate λ_1. The failure of any of the working units in states 1 and 2 brings the system to a completely failed state F. Common cause failure and critical human error can occur in all the three working states which cause complete failure of the system and are denoted by C and H, respectively. A repair facility is available in states 1, 2 and all the completely failed states F, C, and H. After repair, the system goes to state 0. Repair rates from states 1 and 2 follow exponential distributions while from states F, C and H follow general distributions.

By using supplementary variable technique and probability analysis method, the dynamic behavior of the system can be governed by the partial differential equations

$$\begin{cases} (\frac{d}{dt}+\lambda+\lambda_0+\lambda_{h_0}+\lambda_{c_0})p_0(t)=\mu_1p_1(t)+\mu_2p_2(t)+\int_0^\infty\alpha(x)p_F(x,t)dx+\int_0^\infty\beta(y)p_H(y,t)dy+\int_0^\infty\gamma(z)p_C(z,t)dz, \\ (\frac{d}{dt}+\lambda+\mu_1+\lambda_{h_1}+\lambda_{c_1})p_1(t)=\lambda_0p_0(t), \\ (\frac{d}{dt}+\lambda_1+\mu_2+\lambda_{h_2}+\lambda_{c_2})p_2(t)=\lambda p_0(t), \\ (\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\alpha(x))p_F(x,t)=0, \\ (\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\beta(y))p_H(y,t)=0, \\ (\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\gamma(z))p_C(z,t)=0, \end{cases} \tag{1}$$

with the boundary conditions

$$\begin{cases} p_F(0,t)=\lambda p_1(t)+\lambda_1p_2(t), \\ p_H(0,t)=\sum_{j=0}^{2}\lambda_{h_j}p_j(t), \\ p_C(0,t)=\sum_{j=0}^{2}\lambda_{c_j}p_j(t), \end{cases} \tag{2}$$

and initial conditions

$$p_0(0)=1,\ \text{the others equal to } 0, \tag{3}$$

where $p_i(t)$ represents the probability that the system is in state i at time t $(i=0,1,2)$, $p_j(u,t)$ represents the probability density with respect to repair time that the system is in state j and has an elapsed repair time $u\,(u=x,y,z;\,j=F,H,C)$. $\lambda_{c_i}/\lambda_{h_i}$ denotes the constant failure rate from state $i\,(i=0,1,2)$ to state C/H, μ_k denotes the constant repair rate from state $k\,(k=1,2)$ to state 0, and $\alpha(x)/\beta(y)/\gamma(z)$ denotes the repair rate from state $F/H/C$ to state 0 which is nonnegative and locally integrable on $[0,\infty)$ satisfying