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New indicators of chaos

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ABSTRACT

An algorithm which allows to recognize whether the evolution of the system is periodic, chaotic or hyperchaotic is proposed. Minimum length orbit is introduced. Its behavior depends on the kind of solution. The calculations are presented for the Hénon map, for the damped driven pendulum, for the Rössler system, and for the hyperchaotic Qi system.

1. Introduction

A deterministic system can evolve in a way which, in the long term, is unpredictable. The analysis of this kind of evolution is the objective of the theory of chaos. Applications of this theory include physics, bioinformatics, biomedicine, meteorology, chemistry, sociology, astrophysics, engineering, economy. An excellent review of the history of the concepts underlying theory of chaos, from the 17th century to the last decade, has been given by Christophe Letellier in his book "Chaos in nature" [1].

Though a large number of different kinds of indicators of chaos may be found in the literature [2–13] in some areas of science some new and simple concepts are still needed. For example, in order to analyze the motion of the planetary systems, indicators of chaos which could utilize in a simple way the data derived from astronomical observations are of a great importance [14,15]. In this area the control of chaotic systems by time-series analysis [16,17] may be particularly useful. To this family of the indicators of chaos one can assign a simple method based on the statistical properties of the time-series recently formulated by us [18,19].

This kind of studies is continued in the present work. A simple algorithm for the calculation of a value referred to as *the minimum length orbit* is introduced. It is shown that it behaves differently for different kinds of solutions. The studies are performed for four systems. In particular, an analysis of the properties of the recently introduced hyperchaotic *Qi* system [20] are considered. Studies on hyperchaos, which may appear in at least four-dimensional autonomous systems is a subject of many investigations in science and engineering [21–28]. Hyperchaotic systems are characterized by more than one positive Lyapunov exponent. Generally, hyperchaos is more disordered than the ordinary chaos and the new indicator is expected to reveal also this property (see subsequent sections).

2. Method

Let us consider a dynamical system. Its evolution, described by n time-series $x_j(t), j = 1, 2, \dots, n$, corresponds to a trajectory in the appropriate n-dimensional space. Let us assume that the time-series is subject of experimental measurements on a set of L values of time: $\tau_1, \tau_2, \dots, \tau_L$. The aim is to construct an algorithm which allows to recognize whether the measured evolution of the system is periodic, chaotic or hyperchaotic. The periodic motion is by definition self-similar over sufficiently large intervals of time. This means overlapping of the trajectories, and as a consequence a final shape of the trajectory (i.e.

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orbit) is a closed curve. Then, the sum of the intervals between the points in the n-dimensional space (*the length of the orbit*) never exceeds a certain value. In the case of chaos and hyperchaos such a limitation does not exist. The algorithm constructed is based on this observation.

Let $X(\tau_i)$ be a point in the n-dimensional space:

$$X(\tau_i) = \{x_1(\tau_i), x_2(\tau_i), \ldots, x_n(\tau_i)\}. \tag{1}$$

Let us split the set of points

$$S_L = \{X(\tau_i)\}_{i=1}^{L}, \tag{2}$$

to K subsets

$$S_{N_1} \subset S_{N_2} \subset \cdots \subset S_{N_K} = S_L, \tag{3}$$

with

$$S_{N_k} = \{X(\tau_i)\}_{i=1}^{N_k}. \tag{4}$$

Now let us construct the *minimum length orbit* D_k in S_{N_k}. Let us define the interval between a and b in S_{N_k}

$$\Delta_k^{a,b} = \left[\sum_{j=1}^{n} \left[x_j(\tau_a) - x_j(\tau_b)\right]^2\right]^{1/2}. \tag{5}$$

Let us start to calculate the length of the orbit at $X(\tau_{y_0}) \equiv X(\tau_1)$. Then we find the closest point $X(\tau_{y_1})$ to the previous one ($X(\tau_{y_0})$) and calculate

$$\Delta_k^{y_0,y_1} = \min_{i \neq y_0} \left\{\Delta_k^{y_0,i}\right\}. \tag{6}$$

The consecutive point, $X(\tau_{y_2})$, is obtained in a similar way

$$\Delta_k^{y_1,\tau_{y_2}} = \min_{i \neq y_0, y_1} \left\{\Delta_k^{y_1,i}\right\}. \tag{7}$$

The minimum length orbit is defined as

$$D_k = \sum_{i=1}^{N_k - 1} \Delta_k^{y_{(i-1)},y_i}. \tag{8}$$

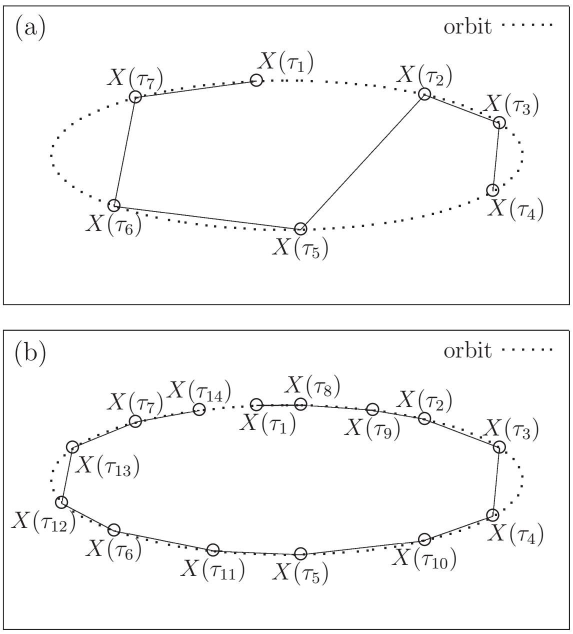

Fig. 1. A model example of the periodic orbit for $n = 2$, $K = 2$. Panel (a): Minimum length orbit D_1. Panel (b): Minimum length orbit D_2.