Applied Mathematics and Computation 227 (2014) 480–493

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

# Hybrid adaptive and impulsive synchronization of uncertain complex networks with delays and general uncertain perturbations

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ARTICLE INFO

*Keywords:*
Complex networks
Synchronization
Uncertainties
Adaptive control
Impulsive control

ABSTRACT

This paper is concerned with the problem of asymptotic synchronization for a class of uncertain complex networks with delays and general uncertain perturbations. In order to cope with the bad effects generated by the uncertain perturbations, a novel hybrid adaptive and impulsive controller is designed such that the complex network can be asymptotically synchronized onto an isolate chaotic system with uncertain perturbations. All the perturbations can be different from each other. On the basis of a new lemma, squeezing rule, and Lyapunov–Krasovskii functionals, several sufficient conditions guaranteeing the realization of the synchronization goal are derived. It is shown that the designed hybrid controllers exhibit powerful robustness. Some existing results are improved and extended. Numerical simulations verify the effectiveness of the theoretical results and the robustness of the new controller.



## 1. Introduction

In recent years, much attention has been payed to the dynamics of complex networks. The main reason is that many real systems can be described by complex dynamical networks, such as the internet networks [1], biological networks [2], epidemic spreading networks [3], collaborative networks [4] and social networks [5], etc.

Synchronization, as an important collective behavior of complex dynamical networks, has received particular attention due to its broad applications in different fields such as secure communication [6], information processing [7]. In the literature, there are many results concerning synchronization of complex dynamical networks with and without delays [8–14]. Along with the study of synchronization, different control techniques have also been developed. For instance, state feedback control, adaptive control, intermittent control, impulsive control, pinning control. Among these control techniques, adaptive control and impulsive are attractive. Adaptive control is an effective control technique in synchronizing coupled chaotic systems due to its good robustness [8,11–15,19]. The advantage of adaptive control is that the control parameters can adjust themselves according to some suitable updating laws, which are designed according to control purpose and characteristics of considered system. By using adaptive control, authors of [8] studied the synchronization for a class of general uncertain coupled complex networks with delays and stochastic perturbations. Impulsive control is attractive since the it acts only at discrete time-points. By using impulsive control technique, control cost and the amount of the transmitted information can

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http://dx.doi.org/10.1016/j.amc.2013.11.025

be reduced drastically [20–24]. Recently, by integrating both the advantages of adaptive and impulsive control methods, the authors of [25,26] investigated the synchronization for a class of uncertain complex dynamical networks. However, the authors of [25,26] did not consider time delay. It is well known that time delays usually exist in spreading due to the finite speeds of transmission as well as traffic congestions. Therefore, it is significant to investigate the synchronization of complex networks with delays.

It is worth noting that, in most of existing papers concerning synchronization of complex networks, all dynamical nodes are assumed to immune from uncertain perturbations. In practice, chaotic systems are inevitably subject to many types of uncertainties, such as unknown parameters, unknown nonlinearities, exogenous disturbances and artificial factors, etc. For instance, it is reported that the famous Lorenz system is derived from partial differential equations after a series of approximations [27]. On the other hand, it is difficult to keep the coupled systems to be identical all the time since the parameters of dynamical nodes may be variant due to environmental changes [28]. Thus, studying synchronization of complex networks with uncertain perturbations is necessary and useful in both theoretical research and practical applications. In [8], synchronization of complex networks subject to random perturbations was studied. In [14,17], synchronization in an array of stochastically coupled networks was studied, However, as every one knows, not all perturbations are stochastic. The common feature of the perturbations in [8,14,17] is that, when synchronization has been realized, the perturbations vanish. However, if nonzero uncertain exogenous inputs are acquired by some nodes in a complex network, then the uncertain perturbations induced by the exogenous inputs can not disappear even all the dynamical nodes synchronize with each other. Recently, based on the passivity property and linearization method, authors of [29] studied the problems of passive control and synchronization of complex networks with and without coupling delay. However, their results were conditioned by the assumption that exogenous input to each node was zero. A natural question is: do complex networks subject to general uncertain perturbations can be controlled to synchronization? This paper will give a positive answer. Our method to deal with synchronization of complex networks with general uncertain perturbations is designing a new adaptive controller, which contains the usual adaptive controller used in [8–19,25,26] as a special case.

To sum up, this paper shall study synchronization for a class of uncertain complex dynamical networks with delays and general uncertainties. In order to synchronize the considered model onto an isolate node system with different uncertain perturbations, new hybrid adaptive and impulsive controller is designed. Several synchronization criteria are derived by using squeezing rule and Lyapunov–Krasovskii functions. The designed adaptive controller includes the usual adaptive controller as a special case. It is shown theoretically and numerically that the hybrid controller exhibits powerful robustness. It can synchronize the considered model onto a smooth trajectory even without knowing priori whether each node in the network is perturbed or not. Some existing results are improved and extended. Numerical simulations verify the effectiveness of the theoretical results.

The rest of this paper is organized as follows. In Section II, the considered model of coupled dynamical networks with uncertain perturbations is presented. Some necessary assumptions, definitions and lemmas are also given in this section. Section III develops several synchronization criteria for the proposed model. Synchronization rate is also estimated for the considered model without delay. Then, in Section IV, a simulation example is presented to show the effectiveness and robustness of the new controller. Finally, Section V reaches some conclusions. Future research field is also discussed in this section.

**Notations:** Symbols in this paper are quite standard. $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean space, the superscript $T$ stands for the transpose of a vector, accordingly, for vector $x \in \mathbb{R}^n$, $\|x\| = x^Tx, \mathbb{R}_+ = [0, +\infty)$, $\mathbb{N}$ denotes the set of natural numbers, $o(y)$ denotes infinitesimal of higher order than function $y$. $\mathrm{sgn}(\cdot)$ is the sign function, $I$ is an identity matrix with appropriate dimension, $\lambda_{\max}(A)$ is the maximum eigenvalue of the matrix $A$.

## 2. Preliminaries

Consider a complex dynamical network consisting of $N$ identical nodes with uncertain couplings and uncertain perturbations, which is described as

$$\begin{aligned}\dot{x}_i(t) = {}& f_1(t, x_i(t)) + f_2(t, x_i(t-\tau_1)) + h_i(x_1(t), x_2(t), \dots, x_N(t)) + g_i(x_1(t-\tau_2), x_2(t-\tau_2), \dots, x_N(t-\tau_2)) \\ & + \sigma_i(t, x_i(t), x_i(t-\tau_3)), \quad i = 1, 2, \dots, N,\end{aligned} \tag{1}$$

where $x_i(t) = [x_{i1}(t), \dots, x_{in}(t)]^T \in \mathbb{R}^n$ represents the state vector of the $i$th node. $f_1$ and $f_2 : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ are continuously differentiable nonlinear vector functions. $\tau_1, \tau_2$ and $\tau_3$ are time delays. $h_i, g_i : \mathbb{R}^{nN} \to \mathbb{R}^n$ are unknown coupling functions. Vector $\sigma_i(t, x_i(t), x_i(t-\tau_3)) = [\sigma_{i1}(t, x_i(t), x_i(t-\tau_3)), \dots, \sigma_{in}(t, x_i(t), x_i(t-\tau_3))]^T \in \mathbb{R}^n$ describes the uncertain perturbations to the $i$th node.

We assume that (1) has a unique continuous solution for any initial condition of the following form: $x_i(s) = \varphi_i(s) \in C([-\tau, 0], \mathbb{R}^n), i = 1, 2, \dots, N,$ where $\tau = \max\{\tau_1, \tau_2, \tau_3\}$, $C([-\tau, 0], \mathbb{R}^n)$ denotes the set of all continuous functions from $[-\tau, 0]$ to $\mathbb{R}^n$.

The delayed dynamics of an isolate node with uncertain perturbation are described by

$$\dot{z}(t) = f_1(t, z(t)) + f_2(t, z(t-\tau_1)) + \overline{\sigma}(t, z(t), z(t-\tau_3)), \tag{2}$$

where $\overline{\sigma}(t, z(t), z(t-\tau_3)) = [\overline{\sigma}_1(t, z(t), z(t-\tau_3)), \dots, \overline{\sigma}_n(t, z(t), z(t-\tau_3))]^T \in \mathbb{R}^n$ describes the uncertain perturbation on the isolate node, and $z(t)$ can be any desired state: equilibrium point, a nontrivial periodic orbit, or even a chaotic orbit.