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Two temperature generalized magneto-thermoelastic interactions in an elastic medium under three theories

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ABSTRACT

Magneto-thermoelastic interactions in an isotropic homogeneous elastic half-space with two temperatures are studied using mathematical methods under the purview of the Lord–Şhulman (LS) and Green–Lindsay (GL) theories, as well as the classical dynamical coupled theory (CD). The medium is considered to be permeated by a uniform magnetic field. The general solution obtained is applied to a specific problem of a half-space and the interaction with each other under the influence of magnetic field subjected to one types of heating the thermal shock type. The normal mode analysis is used to obtain the exact expressions for the displacement components, force stresses, temperature and couple stresses distribution. The variations of the considered variables through the horizontal distance are illustrated graphically. Comparisons are made with the results between the three theories. Numerical work is also performed for a suitable material with the aim of illustrating the results.

1. Introduction

The classical coupled thermoelasticity theory proposed by Biot [1] with the introduction of the strain-rate term in the Fourier heat conduction equation leads to a parabolic-type heat conduction equation, called the diffusion equation. This theory predicts finite propagation speed for elastic waves but an infinite speed for thermal disturbance. This is physically unrealistic. To overcome such an absurdity, generalized thermoelasticity theories have been propounded by Lord and Şhulman [2] as well as Green and Lindsay [3] advocating the existence of finite thermal wave speed in solids. These theories have been developed by introducing one or two relaxation times in the thermoelastic process, either by modifying Fourier's heat conduction equation or by correcting the energy equation and Neuman–Duhamel relation. According to these generalized theories, heat propagation can be visualized as a wave phenomenon rather than a diffusion one; in the literature, it is usually referred to as the second sound effect. These three theories are structurally different from one another and one can not be obtained as a particular case of the other. Various problems characterizing these theories have been investigated and has revealed some interesting phenomenon. Brief reviews of this topic have been reported by Chandrasekharaiah [4,5]. The coupled theory of thermoelasticity has been extended by including the thermal relaxanon time in the constitutive equations by Lord and Shulman [2] and Green and Lindsay [3]. These theories eliminate the paradox of infinite velocity of heat propagation and are termed generalized theories of thermoelasticity. This exist in the following differences between the two theories

1. The Lord–Şhulman theory (L–S) involves one relaxation time of thermoelastic process (τ_0). The Green and Lindsay (G–L) involves two relaxation times (τ_0, ν_0).

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Nomenclature

λ, μ	counterparts of Lame's parameters
p	initial pressure
η	initial stress parameter
a	two temperature parameter
α_t	coefficient of linear thermal expansion
$\theta = T - T_0$	thermodynamical temperature
$\phi = \phi_0 - T$	conductive heat temperature (thermal temperature)
T	absolute temperature
T_0	temperature of the medium in its natural state assumed to be $\lvert \frac{T-T_0}{T_0} \rvert < 1$
σ_{ij}	components of the stress tensor
u_i	components of the displacement vector
ρ	density of the medium
e_{ij}	components of the strain tensor
e	cubical dilatation
C_E	specific heat at constant strain
K	thermal conductivity
τ_0	thermal relaxation time
μ_0	magnetic permeability
ε_0	electric permittivity
F_i	Lorentz force
δ_{ij}	Kronecker delta function

2. The LS energy equation involves first and second time derivatives of strain, whereas the corresponding equation in GL theory needs only the first time derivative of strain.
3. In the linearised case according to the approach of (G–L) theory the heat cannot propagate with finite speed unless the stresses depend on the temperature velocity, whereas according to (L–S) theory the heat can propagate with finite speed even though the stresses there are independent of the temperature velocity.
4. The Lord–Şhulman theory (L–S) can not be obtained from Green and Lindsay (G–L) theory.

The study of the electromagneto-thermoelastic interactions which deals with the interactions among the strain, temperature and the electromagnetic field in an elastic solid is of great practical importance due to its extensive uses in diverse field, such as geophysics (for understanding the effect of the Earth's magnetic field on seismic waves), damping of acoustic waves in a magnetic field, designing machine elements like heat exchangers, boiler tubes where the temperature induced elastic deformation occurs, biomedical engineering (problems involving thermal stress), emissions of the electromagnetic radiations from nuclear devices, development of a highly sensitive super conducting magnetometer, electrical power engineering, plasma physics etc. The interplay of the Maxwell electromagnetic field with the motion of deformable solids is largely being undertaken by many investigators owing to the possibility of its application to geophysical problems and certain topics in optics and acoustics. Moreover, the earth is subject to its own magnetic field and the material of the earth may be electrically conducting. Thus, the magneto-elastic nature of the earth's material may affect the propagation of waves. Many authors have considered the propagation of electro-magnto-thermoelastic waves in an electrically and thermally conducting solid. During the second half of 20th century, great attention has been devoted to the study of electromagneto-thermoelastic coupled problems based on the generalized thermoelasticity. The magneto-thermoelastic disturbances generated by a thermal shock in an elastic half-space having a finite conductivity has been investigated by Puri [6]. Among the authors who considered the generalized magneto-thermoelastic equations are Nayfeh and Nemat-Nasser [7] who studied the propagation of plane waves in a solid under the influence of an electromagnetic field. They have obtained the governing equations in the general case and the solution for some particular cases. Choudhuri [8] extended these results to rotating media. Ezzat [9] has studied the problem of generation of generalized magneto-thermoelastic waves by thermal shock in a perfectly conducting half-space. Ezzat et al. [10] have established the model of two dimensional equations of generalized magneto-thermoelasticity. In dealing with classical or generalized thermoelastic problems in most situations, the displacement potential function approach is used. However Bahar and Hetnarski [11–12] outlined several disadvantages of the potential function approach. These may be summarized in the fact that the boundary and initial conditions of the problem are not related directly to the potential function, as it has no physical meaning explicitly.

Secondly, more stringent assumptions must be made on the behaviour of potential functions than on the actual physical quantities. Last of all, it was found that many integral representations of physical quantities are convergent in the classical sense while their potential function representations only converge in the mean. To get rid of these difficulties, Bahar and Hetnarski [13] introduced the state space formulation in thermoelastic problems. This state space approach has been further