Applied Mathematics and Computation 220 (2013) 20–37

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

The full positive flows of Manakov hierarchy, Hamiltonian structures and conservation laws

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ARTICLE INFO

Keywords:
The full positive flows of Manakov hierarchy
Hamiltonian structure
Conservation laws

ABSTRACT

Based on four sets of Lenard recursion sequences and zero-curvature equation, we derive the full positive flows of the Manakov hierarchy associated with a 3×3 matrix spectral problem, from which some new nonlinear evolution equations are proposed. With the help of the Darboux transformation, soliton solutions of two new nonlinear evolution equations in the Manakov hierarchy are constructed. As two special reductions, the full positive flows of the coupled modified Korteweg–de Vries hierarchy and the Sasa–Satsuma hierarchy are deduced, in which some new nonlinear evolution equations are included. And then, we construct the Hamiltonian structures of the Manakov hierarchy and infinite conservation laws of several nonlinear evolution equations.

1. Introduction

In recent years, the study of coupled nonlinear Schrödinger (CNLS) equations has attracted a great deal of attention due to their appearance as modeling equations in diverse areas including deep water waves, nonlinear fibre optics, plasma physics, etc. Among such systems, one fairly general and frequently studied CNLS equation is of the following dimensionless form [1–3]

$$\begin{aligned} &iu_t + \eta_1 u_{xx} + 2(\mu|u|^2 + \delta|v|^2)u = 0, \\ &iv_t + \eta_2 v_{xx} + 2(\delta|u|^2 + \nu|v|^2)v = 0, \end{aligned} \tag{1.1}$$

which describes the simultaneous propagation of two nonlinear waves in a nonlinear, dispersive medium. Sahadevan et al. [4] confirmed that the system (1.1) possesses Painlevé property under two particular conditions: (1) $\eta_1 = \eta_2, \mu = \nu = \delta$; (2) $\eta_1 = -\eta_2, \mu = \nu = -\delta$. These parametric choices are identical to those of Zakharov and Schulman [5] who established the integrability in terms of motion invariants. After suitable scale transformations, the first case can be transformed into the celebrated Manakov model [6]

$$\begin{aligned} &iu_t + u_{xx} + 2(|u|^2 + |v|^2)u = 0, \\ &iv_t + v_{xx} + 2(|u|^2 + |v|^2)v = 0, \end{aligned} \tag{1.2}$$

which was first proposed by Manakov to describe intense electromagnetic pulse propagation in birefringent fibre. Subsequently, this system was derived as a key model for lightwave propagation in optical fibres [7]. So far, there has been much

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http://dx.doi.org/10.1016/j.amc.2013.05.050

research on the Eq. (1.2). Multi-soliton solutions and collisions were discussed by the method of inverse scattering transformation [1,8], Hirota bilinear approach [9–13], algebraic technique [14], Bäcklund transformation [15], Darboux transformation [16], KP-hierarchy reduction [17] and others [18–29]. Morever, the periodic and quasi-periodic waves solutions for the CNLS equation of Manakov type were presented by using Lax pair method and the general method of reduction of Abelian functions to elliptic functions [30–32]. Elgin et al. [33,34] constructed the finite gap solutions to the vector NLS equation using an algebro-geometric technique. There are some other articles refer to the periodic and quasi-periodic solutions with different methods [35–41]. Furthermore, a new finite-dimensional Hamiltonian system which proved to be completely integrable in the Liouville sense was obtained by nonlinearization [42]. Also, some other integrable properties of the CNLS equation have been investigated, such as, infinitely many local conservation laws [43,44], an infinite-dimensional algebra of non-commutative symmetries [45], Bäcklund transformation [46], Darboux transformation [47], bilinear representation [48].

In this paper, based on the 3×3 matrix spectral problem [49,50], we derive the full positive flows of the Manakov hierarchy and its two reductions, from which some new nonlinear evolution equations are proposed. Besides, we discuss Hamiltonian structure and conservation laws of these nonlinear evolution equations. The present paper is organized as follows: In Section 2, by means of four sets of Lenard recursion equations, we obtain the full Manakov hierarchy with positive power expansions of the spectral parameter. From the first member of the Manakov hierarchy, we give two new nonlinear evolution equations

$$\begin{aligned} &u_{xt}w = u_{xx}w + w_x(u_t - u_x) + uw^2r, \\ &v_{xt}r = v_{xx}r + r_x(v_t - v_x) + vwr^2, \\ &w_{xt}u = 2w_{xx}u + u_x(w_t - 2w_x) - u^2vw, \\ &r_{xt}v = 2r_{xx}v + v_x(r_t - 2r_x) - uv^2r, \end{aligned} \tag{1.3}$$

and

$$\begin{aligned} &u_t = w_x + w\partial^{-1}(uv - wr) - u\partial^{-1}vw, \\ &v_t = v\partial^{-1}vw, \\ &w_t = w\partial^{-1}vw, \\ &r_t = v_x - v\partial^{-1}(uv - wr) - r\partial^{-1}vw, \end{aligned} \tag{1.4}$$

where $\partial = \partial_x, \partial^{-1}\partial = \partial\partial^{-1} = 1$. Then the explicit solutions, including the soliton solutions, to Eqs. (1.3) and (1.4) are obtained with the help of the Darboux transformation. Eqs. (1.3) and (1.4) can be used to describe the soliton wave phenomenon in the nonlinear waves in a nonlinear, dispersive medium (see also the figures of the solutions below). The classical Manakov model (1.2) generates from the second member in it. Then we study the Hamiltonian structure of the Manakov hierarchy and the infinite conservation laws of the first equation in the hierarchy and Eq. (1.2). In Section 3, from one reduced spectral problem, we arrive at the full hierarchy of coupled modified Korteweg–de Vries (CMKdV) equations with positive power expansions of the spectral parameter, certain particular cases in the first member of which are three new nonlinear evolution equations

$$\begin{aligned} &u_{xt}v = v_{xx}v + v_x(u_t - v_x) + v^2(u^2 - v^2), \\ &v_{xt}u = u_{xx}u + u_x(v_t - u_x) - u^2(u^2 - v^2), \end{aligned} \tag{1.5}$$

$$\begin{aligned} &u_{xt}v = u_{xx}v + v_x(u_t - u_x) + uv^3, \\ &v_{xt}u = 2v_{xx}u + u_x(v_t - 2v_x) - u^3v, \end{aligned} \tag{1.6}$$

and

$$\begin{aligned} &u_{xt}v = u_{xx}v + v_x(u_t - u_x) - uv^3, \\ &v_{xt}u = u_xv_t + u^3v. \end{aligned} \tag{1.7}$$

The second member can be reduced to the CMKdV equation [51]

$$\begin{aligned} &u_t = -u_{xxx} + 6u^2u_x + 3u_xv^2 + 3uvv_x, \\ &v_t = -v_{xxx} + 6v^2v_x + 3u^2v_x + 3uu_xv. \end{aligned} \tag{1.8}$$

Next, we investigate conservation laws of the first member in the hierarchy and the CMKdV equation. In Section 4, by the other reduced spectral problem, we deduce the full positive flows of Sasa–Satsuma hierarchy, from which we present two new nonlinear evolution equations