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An efficient test allocation for software reliability estimation

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ABSTRACT

We propose an efficient sampling scheme in the software reliability estimation. In contrast to fixed sampling schemes, where the proportion of test cases taken from each partition is determined before reliability testing begins, we make allocation decisions dynamically throughout the testing process. We then refine these estimates in an iterative manner as we sample. We also compare the result from the accelerated sampling scheme with the best fixed sampling scheme and demonstrate its superiority over the best fixed sampling scheme in terms of the expected loss incurred when the overall reliability is estimated by its Bayes estimator both theoretically and through Monte Carlo simulations.

1. Introduction

Software testing is one of the most important methods to guarantee and improve software reliability [3]. In industry a system often needs to meet a specific level of reliability. Overestimating reliability could have disastrous consequences. Therefore, estimates that closely approximate reality can reduce risk and decrease the cost of software development.

We begin formally by partitioning the test domain into k sub-domains denoted here by $D_1, D_2, \ldots, D_k$. For each sub-domain D_i, we have two associated values: p_i, which is the probability that a given post-delivery use of the software will be of partition i (in other words, $p_1, p_2, \ldots, p_k$ are known parameters of the operational profile [1,23]), and R_i, which is the conditional reliability of a use, on condition that it was randomly chosen from within partition i. Within partition i, each test case has an equal chance of being selected.

Our approach is to break up the domain of possible test cases into partitions. It is required that these partitions be non-overlapping such that if test case i belongs to partition j then no partition other than j will contain test case i. Software testing using samples of test cases drawn from partitions such as this is referred to as partition testing [1,21,24].

The definition of reliability used here will be the one described by Poore et al. [15]: “Reliability is the probability that the software will give the correct result for a single randomly chosen [according to the operational profile] use.” Using this definition, software reliability, R, is represented by:

$$R = \sum_{i=1}^{k} p_i R_i, \tag{1.1}$$

See for example [1,21].

The impossibility of complete testing of any software system of non-trivial size precludes us from knowing the conditional reliability R_i of each sub-domain exactly [9]. Instead, we must distribute the M test cases allocated for reliability estimation among these k partitions, and use the results to estimate each R_i. Specifically, sample sizes $m_1, m_2, \ldots, m_k$ are taken

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from sub-domains $D_1, D_2, \ldots, D_k$ respectively, where $m_1 + m_2 + \cdots + m_k = M$. We take a Bayesian approach that allows us to express an initial belief about the conditional reliability within a particular sub-domain, and refine these beliefs as we sample to improve future allocation decisions.

Bayesian-based allocations are sought by [3,8,12], although the focus here is on estimating software reliability instead of software predictive reliability [3], p-values estimation [8], or estimating the probability of failure [12]. Other criteria such as maximizing the payoff in expected reliability [22] have been studied. For related work on Bayesian-based optimal test allocations in software reliability, see [19].

In the remaining sections of this paper, the Bayesian model and software reliability estimation will be discussed in Sections 2 and 3 followed by a brief introduction to best-fixed sampling scheme in Section 4. We will then propose the sequential allocation and accelerated sampling scheme in Sections 5 and 6. The comparisons between the proposed scheme and best-fixed sampling scheme as well as other schemes are given in the remaining sections with both theoretical results and Monte Carlo simulations.

2. The Bayesian model

We model the outcome of the *jth* test taken from the *ith* partition as a Bernoulli random variable $x_{i,j}$ such that:

$$x_{i,j} = \begin{cases} 1, \text{if test } j \text{ taken from partition } i \text{ is processed correctly} \\ 0, \text{otherwise} \end{cases}$$

where $x_{i,j}$ has a Bernoulli distribution with parameter R_i. Our estimate of R, which is denoted here as $\hat{R}$, based on this allocation can thus be defined as $\hat{R} = \sum_{i=1}^{k} p_i \hat{R}_i$,where $\hat{R}_i$ is our estimate of R_i after m_i tests have been allocated to partition i such that:

$$\hat{R}_i = E[R_i | x_{i,1}, \ldots, x_{i,m_i}]$$

The first step then is to define a loss function that is a quantitative measure of the loss incurred by estimating R by $\hat{R}$. Here we choose the squared error loss as our measure of distance and thus define the loss function as $\ell(R, \hat{R}) = (R - \hat{R})^2$. For the squared error loss function, the Bayes estimator of R is the posterior mean (see [4] pp. 161). We thus seek to measure the expected loss incurred by estimating R by its Bayes estimator, $\hat{R}$. This expected loss, $\Re\,(P)$, which is also referred to as the Bayes Risk is $\Re(P) = E^{R^J}[(R - \hat{R})^2]$, which is the expected loss with respect to R^J, the joint density of R and the observed data. Since $\hat{R}$ is the posterior mean, we can rewrite the expected loss as:

$$\Re(P) = E[Var[R | x_{1,1}, \ldots, x_{1,m_1}, \ldots, x_{k,1}, \ldots, x_{k,m_k}]],$$

which is the expectation with respect to the marginal *probability density function* (p.d.f.) of $x_{1,1}, \ldots, x_{1,m_1}, \ldots, x_{k,1}, \ldots, x_{k,m_k}$ of the posterior variance of R given that the results of the M total test cases have been observed (see [10,11,20]).

Now we seek a form of Eq. (2.1) that is useful in making allocation decisions. Ideally, we would like to find an objective function to minimize in terms of the proportion of tests allocated to each sub-domain, namely $m_1, m_2, \ldots, m_k$. Under the assumption of independence of the priors, and since $R = \sum_{i=1}^{k} p_i R_i$, and $p_1, \ldots, p_k$ are fixed, then:

$$\Re(P) = \sum_{i=1}^{k} p_i^2 E[Var[R_i | x_{i,1}, \ldots, x_{i,m_i}]]. \tag{2.2}$$

For each sub-domain D_i, we assume that R_i, the corresponding conditional reliability follows a *Beta* distribution such that $R_i \sim Beta(\alpha_i^0, \beta_i^0)$. The choice of α_i^0 and β_i^0 can be made based on what we believe to be the reliability of the sub-domain i prior to reliability testing. We refer to this estimate as μ_i. Our degree of certainty in this prediction is expressed through the standard deviation σ_i. The expected value of R_i before testing begins, assuming a *Beta* distribution with parameters α_i^0 and β_i^0 is

$$E[R_i] = \frac{\alpha_i^0}{\alpha_i^0 + \beta_i^0},$$

and similarly, the variance is

$$Var(R_i) = \frac{\alpha_i^0 \beta_i^0}{(\alpha_i^0 + \beta_i^0)^2 (\alpha_i^0 + \beta_i^0 + 1)}.$$

In search of the parameters of the *Beta* distribution we set $E[R_i] = \mu_i$ and $Var(R_i) = \sigma_i^2$ and by combining the two equations and through algebraic manipulation [12] we find $\alpha_i^0 = \frac{\mu_i^2(1-\mu_i) - \sigma_i^2 \mu_i}{\sigma_i^2}$ and $\beta_i^0 = \frac{(\mu_i^2(1-\mu_i) - \sigma_i^2 \mu_i)(1-\mu_i)}{\sigma_i^2 \mu_i}$. This is true given the restriction that $0 < \mu_i < 1$ and $0 < \sigma_i^2 < \mu_i(1 - \mu_i)$. This offers some guidance to the practitioner in that α_i^0 and β_i^0 can be computed directly based on the values of μ_i and σ_i. The property of the estimate of overall reliability is stated in the following theorem and will serve to guide the allocation schemes presented in Sections 4 and 5.