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Due-window assignment problems with unit-time jobs

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ABSTRACT

We study a class of due-window assignment problems. The objective is to find the job sequence and the window starting time and size, such that the total cost of earliness, tardiness and due-window is minimized. The study assumes unit-time jobs, and considers settings of a single machine and of parallel identical machines. Both the due-window starting time and size are decision variables. For the single machine setting, we study a complete set of problems consisting of all possible combinations of the decision variables and four cost factors (earliness, tardiness, due-window size and due-window starting time). For parallel identical machines, we study a complete set of problems consisting of all possible combinations of the decision variables and three cost factors (earliness, tardiness and due-window size). All the problems are shown to be solvable in no more than $O(n^3)$ time, where n is the number of jobs.

1. Introduction

Due-date assignment problems have attracted scholars in the last three decades. These problems reflect settings where the due-date or the delivery time of a product is determined during sales negotiations with the customer. Setting the due-date relatively late facilitates the scheduling problem and reduces earliness and tardiness cost, but often has important revenue consequences. These revenue consequences may consist of price reduction and/or of goodwill losses. Meeting promised delivery dates or due-dates is clearly one of management's primary objectives. We refer the readers to the survey paper of Gordon et al. [1], and to more recent papers such as: Mosheiov and Yovel [2], Liao and Cheng [3], Baykasoglu et al. [4], Li et al. [5], Nearchou [6], Shabtay [7], Gordon and Strusevich [8], Gordon and Tarasevich [9], Mosheiov and Sarig [10], and Koulamas [11].

In recent years, several papers focused on *due-window* assignment, where the assumption is that jobs completed within a given time interval (rather than a time point) are not penalized, whereas the remaining jobs are penalized according to their earliness/tardiness. In these problems the total cost is a function of the earliness and tardiness of the jobs as well as the location and size of the due-window. The general version of the problem (i.e. with general job processing times and job-dependent costs) is well known to be NP-Hard. In fact, even the special case of (i) a single machine, (ii) a zero-size due-window (a due-date), and (iii) symmetric job (earliness/tardiness) weights, was shown to be NP-hard; see Hall and Posner [12].

In this paper we solve due-window assignment problems on a single machine and on parallel identical machines. We consider various combinations of cost components (earliness, tardiness, due-window starting time, and due-window size), and decision variables (window starting time and window size). We focus on the very important special case of unit processing time jobs, which is known to have a wide range of applications in many industrial environments. The importance of scheduling identical jobs to both researchers and practitioners is reflected in the recent two surveys focusing on this topic: Baptiste and Brucker [13] and Kravchenko and Werner [14].

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Minsum and minmax due date assignment problems with unit-time jobs on a single and on parallel identical machines were studied by Mosheiov and Yovel [2]. They focused on minimizing earliness and tardiness costs, and on minimizing earliness/tardiness and due-date costs. The proposed solutions require computational efforts of $O(n^3)$ and $O(n^4)$, respectively, where n is the number of jobs. An improved $O(n^3)$ algorithm for solving the latter problem was published by Tuong and Soukhal [15]. Various *due-window* assignment problems with unit time jobs have been studied by Mosheiov and Oron [16], Janiak [17], Janiak and Winczaszek [18], Janiak et al. [19,20], Mosheiov and Sarig [21], Janiak et al. [22], and Gerstl and Mosheiov [23].

In the current paper, extensions of some of the above problems to settings of a common due-window are studied. First, the class of single machine scheduling problems is studied exhaustively. The objective function assumed in this class contains all four cost components: earliness, tardiness, due-window starting time, and due-window size. Three combinations of decision variables are considered: problems where the window size is the decision variable (for a given due-window starting time, which may be restrictive), problems where the due-window starting time is the decision variable (for a given due-window size), and problems where both are decision variables. We show that some of these problems are trivially solved (in constant time), and the others are reduced to an assignment problem, and are solved in $O(n^3)$ time.

In the second part of the paper, the class of due-window assignment on m parallel identical machines is studied. Here, the three options of decision variables are considered (as above), however, the objective function contains only three cost components: earliness, tardiness, and due-window size. Again, all combinations are studied, and we show that some of the problems are trivial, while the optimal solution of the others is obtained by solving a number of (no more than three) assignment problems. Thus, the total complexity of the problems studied here never exceeds $O(n^3)$. A challenging line of research will be to include the cost of the due-window starting time in the class of problems on parallel machine settings. We refer the reader to a recent paper, Janiak et al. [22], who introduced an $O(n^5/m^2)$ algorithm for this problem.

In Section 2 we present the formulation. Section 3 studies the single machine problems, and Section 4 addresses the parallel machine problems.

2. Formulation

We consider a set of n unit jobs which have to be processed on m parallel identical machines. All n jobs are available at time zero and preemption is not allowed. All the jobs share a common due-window: jobs completed within the window are not penalized, while jobs completed outside the due-window are penalized according to their earliness/tardiness values. The scheduler needs to determine the start time and size of the due-window. Let d_1 and d_2 denote the starting time and the finishing time of the due-window, respectively, and $D = d_2 - d_1$ denotes its size.

Let α_j and β_j denote the earliness unit cost and the tardiness unit cost of job j, respectively ($j = 1, \ldots, n$). For a given schedule, C_j denotes the completion time of job j, $E_j = \max\{d_1 - C_j, 0\}$ denotes the earliness of job j, and $T_j = \max\{C_j - d_2, 0\}$ denotes the tardiness of job j. In addition, for a given schedule, the number of jobs assigned to machine i is denoted by n_i, $i = 1, \ldots, n$. We define γ to be the unit cost of the due-window starting time, and δ to be the unit cost of due-window size. The objective functions may contain any subset of the following four cost components: earliness ($\sum \alpha_j E_j$), tardiness ($\sum \beta_j T_j$), due-window starting time ($\gamma\, d_1$), and due-window size (δD). The decision variables are d_1 and D. We study the complete set of combinations of these cost components and decision variables.

3. Single machine problems

In this section we focus on the single machine problems, and show that they are either solved trivially (in constant time), or reduced to an assignment problem (and solved in $O(n^3)$). The list of the problems (including decision variables, given parameters and complexity) is given in Table 1. (*Comment*: In scheduling problems involving a common given due-date, we distinguish between a non-restrictive and a restrictive due-date (see e.g. Baker and Scudder [24]). Similarly, when the due-window starting time d_1 is a given parameter, we distinguish between two cases: (i) a non-restrictive d_1 (d_1 is sufficiently large to allow all n jobs to be early, i.e. $d_1 \geqslant n$), (ii) a restrictive $d_1 (d_1 < n)$. Note that problems **S2** and **S6** assume a restrictive due-window starting time.)

Table 1
Single-machine due-window assignment problems: decision variables and complexity.

Problem		Decision variables	Given	Complexity	Reference
S1	$1/p_j = 1,\ D/\sum(\alpha_j E_j + \beta_j T_j)$	D	d_1	$O(1)$	Section 3.1
S2	$1/p_j = 1,\ d_1 = d_1^{res},\ D/\sum(\alpha_j E_j + \beta_j T_j)$	D	$d_1 = d_1^{res}$	$O(1)$	Section 3.1
S3	$1/p_j = 1,\ d_1/\sum(\alpha_j E_j + \beta_j T_j)$	d_1	D	$O(n^3)$	Section 3.2
S4	$1/p_j = 1,\ d_1,\ D/\sum(\alpha_j E_j + \beta_j T_j)$	d_1, D	–	$O(1)$	Section 3.1
S5	$1/p_j = 1,\ D/\sum(\alpha_j E_j + \beta_j T_j) + \delta D$	D	d_1	$O(n^3)$	Section 3.3: Janiak and Winczaszek [15]
S6	$1/p_j = 1,\ d_1 = d_1^{res},\ D/\sum(\alpha_j E_j + \beta_j T_j) + \delta D$	D	$d_1 = d_1^{res}$	$O(n^3)$	Section 3.3
S7	$1/p_j = 1,\ d_1,\ D/\sum(\alpha_j E_j + \beta_j T_j) + \delta D$	d_1, D	–	$O(n^3)$	Section 3.4: Janiak and Winczaszek [15]
S8	$1/p_j = 1,\ d_1/\sum(\alpha_j E_j + \beta_j T_j) + \gamma d_1$	d_1	D	$O(n^3)$	Section 3.2
S9	$1/p_j = 1,\ d_1,\ D/\sum(\alpha_j E_j + \beta_j T_j) + \gamma d_1$	d_1, D	–	$O(1)$	Section 3.1
S10	$1/p_j = 1,\ d_1,\ D/\sum(\alpha_j E_j + \beta_j T_j) + \gamma d_1 + \delta D$	d_1, D	–	$O(n^3)$	Section 3.4