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Bifurcation from double eigenvalue for nonlinear equation with third-order nondegenerate singularity

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ABSTRACT

In this paper, the steady state bifurcation from a double eigenvalue for nonlinear equation with singularity being second-order fully degenerate and third-order nondegenerate is investigated. By the normalized Lyapunov–Schmidt reduction method, the precise criteria for the existence and nonexistence of bifurcation and the topological properties of regular bifurcated branches are obtained.

1. Introduction

Let X_1 and X be two banach spaces, and $X_1 \subset X$ be a dense and compact inclusion. The main objective of this article is to give the precise criteria for the existence and nonexistence of bifurcation and the topological properties of regular bifurcated branches for the following nonlinear equation:

$$L_\lambda u + G(u,\lambda) = 0, \tag{1}$$

where $L_\lambda = -A + B_\lambda : X_1 \to X$,
$A : X_1 \subset X$ a linear homeomorphism,
$B_\lambda : X_1 \to X$ a linear compact operator depending continuously on $\lambda \in \mathbf{R}$,
$G(,\lambda) : X_1 \to X$ a C^∞ operator depending continuously on $\lambda \in \mathbf{R}$, satisfying

$$G(u,\lambda) = G_2(u,\lambda) + G_3(u,\lambda) + o(\|u\|^3_{X_1}),$$

where $G_2 : X_1 \times X_1 \to X$ is a two-multiple linear mapping,
$G_3 : X_1 \times X_1 \times X_1 \to X$ is a three-multiple linear mapping, and $G_2(u,\lambda) = G_2(u,u,\lambda),\; G_3(u,\lambda) = G_3(u,u,u,\lambda)$.

The bifurcation theory for nonlinear eigenvalue problems was initiated by Krasnoselskii [3] and Rabinowitz [4]. They give discussion of this problem and obtain the Krasnoselskii theorem and Rabinowitz global bifurcation theorem respectively. All these results prove the existence of a steady state bifurcation when the parameter λ crosses an eigenvalue of the linearized problem with odd multiplicity, but the existence of bifurcation when the parameter crosses an eigenvalue with even multiplicity remains unknown. For some other results concerning eigenvalue bifurcation, see [2,5,6,8] and references therein.

In [9], Ma and Wang establish the existence of bifurcation when the parameter λ crosses an eigenvalue with even multiplicity under additional condition that the singularity be even-order nondegenerate. However, there is no precise criteria for the existence of bifurcation from eigenvalues with even multiplicity if the singularity be odd-order nondegenerate.

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On the other hand, it is interesting to find that there exists bifurcation from double eigenvalue for many equations from science and engineering, of which the singularities are second-order fully degenerate and third-order nondegenerate.

In this paper, we consider the bifurcation of (1) from a double eigenvalue under assumption that the singularity be second-order fully degenerate and third-order nondegenerate. By the normalized Lyapunov–Schmidt reduction method [10], the precise criteria for the existence and nonexistence of bifurcation and the topological properties of regular bifurcated branches are obtained. At the end of this article, as an example to show how to apply our results in specific problems, we apply our results to the stationary equation of the Swift–Hohenberg equation.

2. Preliminaries

2.1. Normalized Lyapunov–Schmidt reduction

Let the eigenvalues (counting multiplicity) of L_λ be given by $\{\beta_1(\lambda), \beta_2(\lambda), \ldots\}$ with $\beta_i(\lambda) \in \mathbf{R}(i = 1, 2, \ldots)$ such that

$$\beta_1(\lambda) = \beta_2(\lambda) \begin{cases} < 0, & \lambda < \lambda_0, \\ = 0, & \lambda = \lambda_0, \\ > 0, & \lambda > \lambda_0, \end{cases} \tag{2}$$

$$\beta_j(\lambda_0) \neq 0, \quad \forall j \geqslant 3,$$

and $\{e_1(\lambda), e_2(\lambda), \ldots\},\ \{e_1^*(\lambda), e_2^*(\lambda), \ldots\}$ be the eigenvectors of L_λ and L_λ^*, respectively, corresponding to $\beta_i(\lambda)$, such that

$$\begin{aligned} &L_{\lambda_0} e_k(\lambda) = 0, \quad L^*_{\lambda_0} e^*_k(\lambda) = 0, \quad k = 1, 2, \\ &< e_i(\lambda), e^*_j(\lambda) >= \delta_{ij}, \quad \forall i, j. \end{aligned} \tag{3}$$

By the Spectral Theorem (see [10]), in the neighborhood of $\lambda = \lambda_0,\ X_1$ and X can be decomposed into

$$\begin{cases} X_1 = E_1^\lambda \oplus E_2^\lambda, \\ X = E_1^\lambda \oplus \overline{E}_2^\lambda, \\ E_1^\lambda = span\{e_1(\lambda), e_2(\lambda)\}, \\ E_2^\lambda = \{u \in X_1 \mid < u, e_i^*(\lambda) >= 0, \quad i = 1, 2\}, \end{cases}$$

and L_λ can be decomposed into

$$\begin{cases} L_\lambda = \mathcal{L}_\lambda^0 \oplus \mathcal{L}_\lambda, \\ \mathcal{L}_\lambda^0 : E_1^\lambda \to E_1^\lambda, \\ \mathcal{L}_\lambda : E_2^\lambda \to \overline{E}_2^\lambda. \end{cases}$$

Since $u \in X_1$, there exists $x \in E_1^\lambda,\ y \in E_2^\lambda$, such that

$$u = x + y,$$

$$x = x_1 e_1 + x_2 e_2,$$

$$y = \sum_{j=1}^{\infty} y_j e_j,$$

(1) can be rewritten as follows

$$\beta_1 x_1 + < G_2(x+y) + G_3(x+y), e_1 > + o(\|x+y\|^3_{X_1}) = 0, \tag{4}$$

$$\beta_2 x_2 + < G_2(x+y) + G_3(x+y), e_2 > + o(\|x+y\|^3_{X_1}) = 0, \tag{5}$$

$$\beta_j y_j + < G_2(x+y) + G_3(x+y), e_j > + o(\|x+y\|^3_{X_1}) = 0, \quad j \geqslant 3. \tag{6}$$

Definition 2.1 (*Ma and Wang [10]*). Assume (2) holds true. The steady state solution $u = 0$ of (1) is called kth-order nondegenerate, if $x = (x_1, \ldots, x_r) = 0$ is an isolated singular point of the following system of r-dimensional algebraic equations

$$\sum_{j_1 \cdots j_k = 1}^{r} a^i_{j_1 \cdots j_k} x_{j_1} \ldots x_{j_k} = 0, \quad 1 \leqslant i \leqslant r,$$

where $a^i_{j_1 \cdots j_k} =< G_k(e_{j_1}, \ldots, e_{j_k}),\ e_i^* >_X,\ r$ is the geometric multiplicity of the eigenvalue $\beta_1(\lambda_0)$.