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Mixed fuzzy ideal topological spaces

Binod Chandra Tripathy [a], Gautam Chandra Ray [b,*]

[a] Mathematical Sciences Division, Institute of Advanced Study in Science and Technology, Pashim Boragaon, Guwahati 781035, Assam, India
[b] Department of Mathematics, Central Institute of Technology, Kokrajhar 783370, Assam, India

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ABSTRACT

The aim of this paper is to introduce a new concept of mixed fuzzy ideal topological spaces and investigate some properties of this space.

1. Introduction

The concept of ideal in topological spaces has been introduced and studied by Jankovic and Hamlett [3], Kuratouski [4], Nasef and Mahmoud [7], Vaidyanathswamy [21], Yuksel, Caylak and Acikgoz [23], Yuksel, Acikgoz and Noiri [24] and Yuksel, Kara and Acikgoz [25]. In 1997, Sarkar [9] and Mahmoud [5] introduced the notions of fuzzy ideal and fuzzy local function in fuzzy set theory. Malakar [6] introduced the concepts fuzzy semi-irresolute and strongly irresolute functions. Hatir and Jafari [2] and Nasef and Hatir [8] defined fuzzy semi-*I*-open set and fuzzy pre-*I*-open set via fuzzy ideal. The notion of Ideal has been applied in sequence spaces and different classes of ideal convergent sequences have been introduced and investigated by Tripathy and Dutta [10], Tripathy and Hazarika [11–15], Tripathy and Mahanta [16], Tripathy and Sarma [19], Tripathy, Sen and Nath [20] and many others in the recent years. In 1995, Das and Baishya [1] introduced the concept of mixed fuzzy topological spaces. Tripathy and Ray [17] introduced and studied the concept of mixed fuzzy topological spaces in slightly different ways which is not fuzzification of classical mixed topology. Tripathy and Ray [18] introduced and investigated different properties of fuzzy weakly continuous functions, fuzzy δ-continuous function between two mixed fuzzy topological spaces.

2. Preliminaries and definitions

Let X be a non-empty set and I, the unit interval [0,1]. A fuzzy set A in X is characterised by a function $\mu_A: X \to I$ where μ_A is called the membership function of A and $\mu_A(x)$ representing the membership grade of x in A. The empty fuzzy set is defined by $\mu_\emptyset(t) = 0$ for all $t \in X$. Also X can be regarded as a fuzzy set in itself defined as $\mu_X(t) = 1$ for all $t \in X$. Further, an ordinary subset A of X can also be regarded as a fuzzy set in X if its membership function is taken as usual characteristic function of A that is $\mu_A(t) = 1$ for all $t \in X$ and $\mu_A(t) = 0$ for all $t \in X$-A. Two fuzzy sets A and B are said to be equal if $\mu_A = \mu_B$. A fuzzy set A is said to be contained in a fuzzy set B, written as$A \subseteq B$, if $\mu_A \leqslant \mu_B$. Complement of a fuzzy set A in X is a fuzzy set denoted by A^c in X. Its membership function is defined by $\mu_{A^c} = 1 - \mu_A$.. We can also denote the complement of A by co$A$ that is A^c = co$A$. Union and intersection of a collection $\{A_i : i \in I\}$ of fuzzy sets in X, are written as $\underset{i=1}{\cup} A_i$ and $\underset{i=1}{\cap} A_i$, respectively. The membership functions are defined as follows:

* Corresponding author.
E-mail addresses: gautomofcit@gmail.com, gautam_math@yahoo.com (G.C. Ray).

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$$\mu_{\underset{i\in I}{\cup}A_i}(x) = \sup\{\mu_{A^i}(x) : i \in I\} \quad \text{for all } x \in X.$$

$$\text{and } \mu_{\underset{i\in I}{\cap}A_i}(x) = \inf\{\mu_{A^i}(x) : i \in I\} \quad \text{for all } x \in X.$$

A fuzzy topology τ on X is a collection of fuzzy sets in X such that $\emptyset, X \in \tau$; if $A_i \in \tau$, $i \in I$ then $\underset{i\in I}{\cup} A_i \in \tau$ and if $A, B \in \tau$ then $A \cap B \in \tau$. The pair (X, τ) is called a fuzzy topological space (fts). Members of τ are called open fuzzy sets and the complement of an open fuzzy set is called a closed fuzzy set. If (X, τ) is a fts then the closure and interior of a fuzzy set A in X, denoted by *cl* A and *int* A, respectively, are defined as *cl* $A = \cap\{B$: B is a closed fuzzy set in X and $A \subseteq B\}$ and *int* $A = \cup\{V$: V is an open fuzzy set in X and $V \subseteq A\}$. Clearly, *cl* A (respectively *int* A) is the smallest (respectively largest) closed (respectively open) fuzzy set in X containing (respectively contained in) A. If there is more than one topology on X, then the closure and interior of A with respect to a fuzzy topology τ on X will be denoted by τ-*cl* A and τ-*int* A.

Definition 2.1. A collection B of open fuzzy sets in fts X is said to be an open base for X if every open fuzzy sets in X is a union of members of B.

Definition 2.2. If A is a fuzzy set in X and B is a fuzzy set in Y then, $A \times B$ is a fuzzy set in $X \times Y$ defined as $\mu_{A\times B}(x,y) = \min\{\mu_A(x), \mu_B(y)\}$ for all $x \in X$ and for all $y \in Y$.

Definition 2.3. Let f be a function from X into Y. Then for each fuzzy set B in Y, the inverse image of B under f, written as $f^{-1}[B]$, is a fuzzy set in X defined as $\mu_{f^{-1}[B]}(x) = \mu_B(f(x))$ for all $x \in X$.

Definition 2.4. A fuzzy set A in a fuzzy topological space (X, τ) is called a neighbourhood of a point $x \in X$ if and only if there exists $B \in \tau$ such that $B \subseteq A$ and $A(x) = B(x) > 0$.

Definition 2.5. A fuzzy point x_α is said to be quasi-coincident with A, denoted by $x_\alpha\, qA$, if and only if $\alpha + A(x) > 1$ or $\alpha > (A(x))^c$.

Definition 2.6. A fuzzy set A is said to be quasi-coincident with B and is denoted by AqB, if and only if there exists an $x \in X$ such that $A(x) + B(x) > 1$.

It is clear that if A and B are quasi-coincident at x both $A(x)$ and $B(x)$ are not zero at x and hence A and B intersect at x.

Definition 2.7. A fuzzy set A in a fts (X, τ) is called a quasi-neighbourhood of x_λ if and only if $A_1 \in \tau$ such that $\overline{A_1} \subseteq A$ and $x_\lambda\, qA_1$. The family of all Q-neighbourhood of x_λ is called the system of Q-neighbourhood of x_λ. Intersection of two quasi-neighbourhood of x_λ is a quasi-neighbourhood.

Definition 2.8. Let (X, τ_1) and (X, τ_2) be two fuzzy topological spaces. Consider the collection of fuzzy sets $\tau_1(\tau_2) = \{A \in I^X$: For any fuzzy set B in X with AqB, there exists τ_2-open set A_1 such that A_1qB and τ_1-closure $\overline{A_1} \subseteq A\}$. Then this family of fuzzy sets will form a topology on X and this topology we call a mixed fuzzy topology on X.

Definition 2.9. A non-empty collection of fuzzy sets I of a set X is called a fuzzy ideal if the following postulates are satisfies

(*i*) if $A \in I$ and $B \leqslant A$ then $B \in I$. (heredity)

and (*ii*) if $A, B \in I$ then $A \vee B \in I$. (finite additivity)

The triplet (X, τ, I) is called Ideal fuzzy topological space with the ideal I and fuzzy topology τ.

Definition 2.10. Let A be any subset of X in a fuzzy ideal topological space (X, τ, I). The fuzzy local function of A with respect to τ and I is denoted by $A^*(\tau, I)$ in short A^*. The fuzzy local function $A^*(\tau, I)$ of A is the union of all fuzzy points x_λ such that $U \in N_q(x_\lambda)$ and $E \in I$, then there is at least one $y \in X$ for which $U(y) + A(y) - 1 > E(y)$.

i.e. $A^* = \vee\{x \in X$: $A \vee U \notin I$ for every $U \in \tau(x)\}$.

The fuzzy closure operator of a fuzzy set A in (X, τ, I) is defined as $cl^*(A) = A \vee A^*$. In (X, τ, I) the collection $\tau^*(I)$ means an extension of fuzzy topological space than τ via fuzzy ideal which is constructed by considering the class $\beta = \{U - E$: $U \in \tau$, $E \in I\}$ as a base. A fuzzy subset A of a fuzzy ideal topological space (X, τ, I) is said to be I-open (respectively α-I-open fuzzy set, pre-I-open fuzzy set, semi-I-open fuzzy set, β-I-open fuzzy set) if $A \leqslant int(cl^*(intA))$ (respectively $A \leqslant int(cl^*(int\ (A)))$, $A \leqslant int(cl^*(A))$, $A \leqslant cl^*(int(A))$, $A \leqslant cl(int(cl^*(A))))$.

Definition 2.11. A fuzzy subset A of a fuzzy ideal topological space (X, τ, I) is said to be fuzzy $*$-perfect if $A = A^*$.