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Adaptive synchronization in complex dynamical networks with coupling delays for general graphs

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ABSTRACT

A sufficient condition of adaptive synchronization in dynamical networks with coupling delays is gained, where the coupling configuration corresponds to a weighted graph. One can reduce edges to original graph such that the network more quickly achieves synchronization. Numerical simulations are given to illustrate the efficiency of theoretical results.

1. Introduction

In general, complex networks consist of a large number of nodes and link among them, in which a node is a fundamental cell with specific activity. So complex networks and graphs are closely contacted each other. The dynamics on complex networks is one on graphs, though the graphs may have different characteristics, e.g. classical random graph model [1], small-world model [2,3], scale-free model [4], or others are related closely to natural structure.

Synchronization in complex dynamical networks is a universal phenomenon in various fields of science and society. There are many works on the synchronization in complex dynamical networks [5–9]. Due to the finite speeds of transmission and spreading as well as traffic congestion, a signal or influence traveling through a network often is associated with time delays. Real-world complex systems, particularly in biological and physical systems, are time-delay systems. Thus in recent years, a lot of efforts have been made to study the synchronization of dynamical coupled systems with delays [10–18].

Underlying these researches imply that the structural properties of a network must have some bearing on the synchronization [19,20]. In addition, as pointed in [20], Lu and Cao introduced an adaptive synchronization method by enhancing the coupling strength automatically under a simple updated law. However, their work is limited to tree-like networks (In fact, a tree is a graph without cycles.), which cannot be applied to general networks, and delay effect on synchronization is also unconsidered. In this paper, we will propose an adaptive synchronization method for general networks or graphs with coupling time-delays. Based on the invariant principle of functional differential equations, the global synchronization will be realized by designing adaptive controllers. Finally, the numerical simulations are given to illustrate our theoretical results.

2. Preliminaries

In this section, we now introduce some notations and preliminaries. Consider the delay complex dynamical network consisting of N linearly and diffusively coupled identical nodes, with full diagonal coupling, and each node is an n-dimensional dynamical oscillator which can be chaotic. The state equations of the network are

$$\dot{x}_i = f(x_i) + \sum_{j=1}^{N} a_{ij}\Gamma x_j(t-\tau) + u_i(t), \quad i = 1, 2, \ldots, N, \tag{1}$$

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where $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})^T \in R^n$ is a state vector of node i, $f(x_i) = (f_1(x_i), f_2(x_i), \ldots, f_n(x_i))^T : R^n \to R^n$ is a given nonlinear vector valued function describing the dynamics of the nodes, $u_i(t)$ represents adaptive controller to be designed, and the inner coupling link matrix is a diagonal matrix $\Gamma = \mathrm{diag}\{r_1, r_2, \ldots, r_n\}$ with $r_i > 0$ and $\tau > 0$ is the coupling time-delay. The coupling configuration matrix $A = (a_{ij})_{N\times N}$ is a zero row sums matrix with nonnegative off-diagonal entries, representing the topological structure of the networks.

There is a weighted graph corresponding to the coupling configuration matrix A, called the coupling configuration graph, defined as a graph G on vertices $1, 2, \ldots, N$ which contains an edge ij $(i \neq j)$ with weight a_{ij} if and only if $a_{ij} > 0$. Giving an arbitrary orientation of the edges of G so that each edge has a head and a tail, and a labeling of edges as $e_1, e_2, \ldots, e_E$, where E denotes the number of edges of G. We obtain an edge-vertex incidence matrix of G, denoted as $M := M(A) = (m_{ij})_{E\times N}$, which is defined as $m_{ij} = \sqrt{a_{ij}}$ (reps. $m_{ij} = -\sqrt{a_{ij}}$) if the edge e_i has the vertex j as a head (resp. a tail), and $m_{ij} = 0$ otherwise. The Laplacian matrix of G is defined as $L = M^TM = D - B$, where D is a diagonal matrix and the ith diagonal entry of which is exactly the degree of the vertex i, i.e., $d_i = \sum_{j\in N(i)} a_{ij}$, where $N(i)$ is the set of neighbors of i in the graph G (or the vertices joining i by edges), and $B = (b_{ij})$ is a weighted adjacency matrix of G such that $b_{ij} = a_{ij}$ if ij is an edge of G and $b_{ij} = 0$ otherwise. One can find that $L = -A$ and is symmetric and positive semidefinite, so that its eigenvalues can be arranged as

$$0 = \lambda_0 \leqslant \lambda_1 \leqslant \lambda_2 \leqslant \cdots \leqslant \lambda_{N-1},$$

where $\lambda_0 = 0$ as L has zero row sums, and $\lambda_1 > 0$ if and only if G is connected, and is called the algebraic connectivity of G by Fiedler [21] in the case of G is simple (i.e. all edges have weight 1). If G is connected, the corresponding eigenvector of the eigenvalue 0 is an all ones vector (up to a scalar multiple), denoted by $\mathbf{1}$. One can refer to Chung [22] and Merris [23] for the details of Laplacian matrices of graphs.

If we denote $x = (x_1^T, x_2^T, \ldots, x_N^T)^T$, $F(x) = (f^T(x_1), f^T(x_2), \ldots, f^T(x_N))^T$, $u = (u_1^T, u_2^T, \ldots, u_N^T)^T$ and substitute $-L$ for A, then Eq. (1) is transformed as

$$\dot{x} = F(x) - L \otimes \Gamma x(t-\tau) + u. \tag{2}$$

In this paper, we adopt the l_2-norm for vectors and the induced spectral norm for matrices. We always suppose that the function F in Eq. (2) is Lipschitz continuous, or equivalently f in Eq. (1) is Lipschitz continuous, i.e., there exists a constant $l > 0$ such that for any x, $y \in R^n$,

$$\|f(x) - f(y)\| \leqslant l \cdot \|x - y\|.$$

Lemma 1 [24]. *For any vectors x, $y \in R^n$ and $\varepsilon > 0$, inequality $2x^Ty \leqslant \varepsilon x^Tx + \frac{1}{\varepsilon} y^Ty$ holds.*

Lemma 2 [9]. *Suppose that the coupling configuration graph corresponding to $A = -L$ is connected. Then the dynamical network (2) achieves synchronization if and only if $\lim_{t\to\infty} \|\mathbf{M}x\| = 0$, where $\mathbf{M} = M \otimes I_n$.*

3. Main results

In this section, we will use state feedback control method and invariant principle to investigate adaptive synchronization of complex network (2). To achieve synchronization, we design the adaptive controllers as:

$$u = -k(t)(I_N \otimes \Gamma)x, \tag{3}$$

where $k(t)$ is the time-varying gain. To guarantee negative feedback, the adaptive gain is designed as:

$$\dot{k}(t) = \beta x^T (L \otimes \Gamma)x, \tag{4}$$

where β is a positive constant to be determined.

The main result of this paper is stated as follows:

Theorem 1. *Suppose that F is Lipschitz continuous and the coupling configuration graph corresponding to $A = -L$ is connected. Then the system (2) achieves synchronization under adaptive controllers (3) and (4).*

Proof. Construct a Lyapunov functional as:

$$V(t) = \|\mathbf{M}x(t)\|^2 + \frac{1}{\beta}(k(t) - h)^2 + \int_{t-\tau}^{t} \|\mathbf{M}x(s)\|^2 ds, \tag{5}$$

where h is a sufficiently large constant whose range is given later.

Noting that $\mathbf{M}^T\mathbf{M} = L \otimes I_n$, and substituting Eqs. (3) and (4) for u, we get the derivative of $V(t)$ along the trajectories of Eq. (2) as follows: