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The complete solution procedures for the mathematical analysis of some families of optimal inventory models with order-size dependent trade credit and deterministic and constant demand

Kun-Jen Chung [a], Shy-Der Lin [b], H.M. Srivastava [c,*]

[a] Department of Information Management, Chung Yuan Christian University, Chung-Li 32023, Taiwan, ROC
[b] Departments of Applied Mathematics and Business Administration, Chung Yuan Christian University, Chung-Li 32023, Taiwan, ROC
[c] Department of Mathematics and Statistics, University of Victoria, Victoria, British Columbia V8W 3R4, Canada

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ABSTRACT

By suitably combining the investigations by Ghare and Schrader (1963) [5], Dolan (1987) [4] and Huang and Chung (2003) [7], Kreng and Tan (2011) [8] consider and analyze the optimal inventory policies with order-size dependent trade credit under delayed payment and cash discount. The mathematical analysis of Kreng and Tan (2011) [8] is based upon an inventory model for deteriorating items with trade credit and cash discount linked to the order quantity. Motivated by the potential for practical applications of such inventory models as those that are considered in (for example) the aforecited works, we address some shortcomings in the 2011 paper by Kreng and Tan (2011) [8]. We emphasize upon the invalidity of an important assumption by Kreng and Tan (2011) [8], namely that the deterioration rate is small, provide a counterexample to Kreng and Tan's Theorem 1 and question the results of Kreng and Tan's Theorems 2 and 3. We present our own observations and results as theorems and proofs. We thus have not only removed the aforementioned shortcomings in the paper by Kreng and Tan (2011) [8], but we have also provided the complete solution procedures for some of the aforementioned models. Finally, some numerical examples are used to compare the results, which are presented in this paper, with those of the aforecited earlier investigations.

1. Introduction and motivation

An inventory problem consists of two parts: The modeling part and the mathematical solution procedure part. Basically, both parts are equally important. In recent years, marketing researchers and practitioners have recognized the phenomenon that the supplier offers a permissible delay to the retailer if the outstanding amount is paid within the permitted fixed settlement period, known as the *trade credit period*. During the trade credit period, the retailer can accumulate revenues by selling items and earning interests. As a result, with no incentive for making early payments and earning interest through the accumulated revenue received during the credit period, the retailer postpones payment up to the last moment of the permissible period allowed by the supplier. Therefore, offering trade credit leads to delayed cash inflow and increases the risk of cash flow shortage and bad debt. From the viewpoints of suppliers, they always hope to be able to find a trade credit policy to increase sales and decrease the risk of cash flow shortage and bad debt. In reality, on the operations management side, a supplier is always willing to provide the retailer either a cash discount or a permissible delay in payments.

* Corresponding author.
E-mail addresses: kjchung@cycu.edu.tw (K.-J. Chung), shyder@cycu.edu.tw (S.-D. Lin), harimsri@math.uvic.ca (H.M. Srivastava).

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Goyal [6] establishes a single-item inventory model under permissible delay. Huang and Chung [7] extend the work of Goyal [6] by considering the optimal replenishment and payment polices in the EOQ model under cash discount and trade credit. In addition, Ghare and Schrader [5] first analyze the decaying inventory problem to develop an economic order quantity model. Subsequently, Dolan [4] examines the motivations for suppliers to offer quantity discounts and provides several in-depth examples including pricing for Sealed Air Corporation's protection packing and prices for advertising in the Los Angeles Times. By suitably combining the investigations by Ghare and Schrader [5], Dolan [4] and Huang and Chung [7], Kreng and Tan [8] consider and analyze the inventory model for deteriorating items with trade credit and cash discount linked to the order quantity. Basically, their inventory models are interesting. However, in this paper, we challenge the validity of an important assumption, which is made by Kreng and Tan [8], namely that the deterioration rate is small, provide a counterexample to Kreng and Tan's Theorem 1 and question the results of Kreng and Tan's Theorems 2 and 3. We present our observations and results as theorems and proofs. We thus have not only removed the aforementioned shortcomings in the paper by Kreng and Tan [8], but we have also provided the complete solution procedures for some of the aforementioned models. Finally, some numerical examples are used to compare the results, which are presented in this paper, with those of the earlier investigations.

For an adequate literature survey and review concerning this paper, the reader can refer to the interesting work by Kreng and Tan [8] (see also [10]). In passing, however, we also mention the fundamental work by Stokes [11] who indicated that trade credit represents one of the most flexible sources of short-term financing available to firms principally because it arises spontaneously with the firm's purchases. The decision to trade credit and the determination of the firm's terms of sale are important managerial considerations. In addition, the purchasing firm's decision to take advantage of a cash discount or not and the motivations behind such a decision are also important. Furthermore, given the increasing saliency of a sales promotion tool, Arcelus et al. [1] analyzed the advantages and disadvantages of the two most common payment reduction schemes, namely, a cash discount and a delay in the payment of the merchandise. A cash discount can encourage the customer who pays cash on delivery and reduces the default risk. A permissible delay in payments is considered as a type of price reduction and it can naturally attract new customers and increase sales.

It should be remarked in passing that the various inventory models, which we have investigated in this paper, assume the demand to be deterministic and constant. Nevertheless, in light of several aforementioned in-depth examples including pricing for Sealed Air Corporation's protection packing and prices for advertising in the Los Angeles Times, their complete solution procedure which we are presenting in this paper is believed to have the potential for applications, especially after having satisfactorily removed the shortcomings in the earlier inventory models that are referred to above.

Our presentation here has been organized as follows. In Section 2, we lay out the notations, assumptions and conventions which are used in our investigation. Section 3 contains a rather brief description of the model used by Kreng and Tan [8]. As a part of the optimizing techniques used in this paper (see, for details, [10,12]), in Section 4 we obtain the derivatives of the annual total relevant costs $TC_{ij}(T)$ of the first and the second orders with respect to their argument T in order to ascertain the convexity properties of $TC_{ij}(T)$ and also of several closely-related functions. Our preliminary results are stated here as Lemmas 1 and 2 and are proved in Appendices I and II, respectively. From the results derived in Section 4, we deduce in Section 5 a number of convexity and monotonicity properties of a family of the Δ-functions which were introduced by Kreng and Tan [8]. The convexity results for the Δ-functions are presented systematically as Lemma 3 which is proven in Appendix III. The monotonicity properties of the Δ-functions are presented as Lemma 4 which is proven in Appendix IV. Section 6 contains our main results for the optimal cycle time T^* of the annual total relevant cost $TC(T)$, which we state here as Theorems 1 to 3. The complete demonstration of Theorem 1 is given in Appendix 5 in which we also indicate the corresponding derivations of the results asserted by Theorems 2 and 3. In Section 7, we discuss several illustrative examples with a view to comparing and contrasting our results with the results of Kreng and Tan [8]. Finally, in the concluding section (Section 8), we list a number of remarks and observations which pertain to the various arguments that are put forth in this paper.

2. Notations, assumptions and conventions

Throughout our present investigation, we make use of the followings notations, conventions and assumptions.

Notations Used:

c	unit purchasing cost (\$/unit);
A	ordering cost (\$/order);
h	unit stock holding cost, excluding capital opportunity cost (\$/unit/year);
I_e	annual simple interest rate that can be earned (\$/\$/year);
I_p	annual simple interest charges for an inventory item (\$/\$/year);
α	cash discount rate $(0 < \alpha < 1)$;
T	replenishment cycle time (in years);
D	annual demand;
Q	order size;
s	unit selling price (\$/unit);
T^*	the (fixed) optimal cycle time of $TC(T)$;