Applied Mathematics and Computation 217 (2011) 5743–5753

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Travelling wave solutions of two nonlinear evolution equations by using the $\left(\frac{G'}{G}\right)$-expansion method

Bin Zheng

School of Science, Shandong University of Technology, Zibo 255049, People's Republic of China

ARTICLE INFO

Keywords:
$\left(\frac{G'}{G}\right)$-expansion method
Travelling wave solutions
Integrable sixth-order Drinfeld–Sokolov–Satsuma–Hirota equation
(2+1) dimensional Boussinesq and Kadomtsev–Petviashvili equation

ABSTRACT

By using the $\left(\frac{G'}{G}\right)$-expansion method proposed recently, we give the exact travelling wave solutions of two different types of nonlinear evolution equations in this paper. The travelling wave solutions are expressed by three types of functions. The computation for the method appears to be easier and faster by general mathematical software.

1. Introduction

The nonlinear phenomena exist in all the fields including either the scientific work or engineering fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics, and so on. It is well known that many non-Linear evolution equations are widely used to describe these complex phenomena. So, powerful and efficient methods to find analytic solutions of nonlinear equations have drawn a lot of interest by a diverse group of scientists, and many powerful methods for constructing exact solutions of nonlinear evolution equations have been established and developed such as the inverse scattering transform, the Backlund/ Darboux transform, the tanh-function expansion and its various extension, the Jacobi elliptic function expansion, the homogeneous balance method, the sine–cosine method, the rank analysis method, the exp-function expansion method and so on [1–17], but there is no unified method that can be used to deal with all types of nonlinear evolution equations.

In [18], Mingliang Wang proposed a new method called $\left(\frac{G'}{G}\right)$-expansion method. The value of the $\left(\frac{G'}{G}\right)$-expansion method is that one can treat nonlinear problems by essentially linear methods. The method is based on the explicit linearization of nonlinear evolution equations for traveling waves with a certain substitution which leads to a second-order differential equation with constant coefficients. Moreover, it transforms a nonlinear equation to a simple algebraic computation. The main merits of the $\left(\frac{G'}{G}\right)$-expansion method over the other methods are that it gives more general solutions with some free parameters and it handles in a direct manner with no requirement for initial boundary condition or initial trial function at the outset. Recently, several researchers have studied some nonlinear evolution equations by this method [19–22]. Our goal of this paper is to find out the travelling wave solutions of two known nonlinear evolution equations by using the method.

Drinfeld and Sokolov [23] and independently, Satsuma and Hirota [24] presented the system of coupled nonlinear evolution equations called the integrable sixth-order Drinfeld–Sokolov–Satsuma–Hirota equation. Authors in [25–27] derived the (2+1) dimensional Boussinesq and Kadomtsev–Petviashvili equation. They gave the exact solutions of these two types of equations by other method, while we can obtain the travelling wave solutions of these two types of equations by using $\left(\frac{G'}{G}\right)$-expansion method much easier and faster.

E-mail address: zhengbin2601@126.com

doi:10.1016/j.amc.2010.12.052

The rest of the paper is organized as follows. In Section 2, we describe the $(\frac{G'}{G})$-expansion method for finding travelling wave solutions of nonlinear evolution equations, and give the main steps of the method. In the subsequent sections, we illustrate the method in detail with the integrable sixth-order Drinfeld–Sokolov–Satsuma–Hirota equation and the (2+1) dimensional Boussinesq and Kadomtsev–Petviashvili equation. In the last Section, the features of the $(\frac{G'}{G})$-expansion method are briefly summarized.

2. Description of the $(\frac{G'}{G})$-expansion method

In this section we describe the $(\frac{G'}{G})$-expansion method for finding travelling wave solutions of nonlinear evolution equations. Suppose that a nonlinear equation, say in two independent variables x, t, is given by

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0,$$

or three independent variables x, y and t, is given by

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xx}, u_{yy}, \ldots) = 0, \tag{2.1}$$

where $u = u(x,y,t)$ is an unknown function, P is a polynomial in $u = u(x,t)$ or $u = u(x,y,t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following we give the main steps of the $(\frac{G'}{G})$-expansion method.

Step 1. Suppose that

$$u(x,t) = u(\xi), \quad \xi = x - ct,$$

or

$$u(x,y,t) = u(\xi), \quad \xi = \xi(x,y,t), \tag{2.2}$$

the travelling wave variable (2.2) permits us reducing Eq. (2.1) to an ODE for $u = u(\xi)$

$$P(u, u', u'', \ldots) = 0. \tag{2.3}$$

Step 2. Suppose that the solution of ODE (2.3) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi) = \alpha_m \left(\frac{G'}{G}\right)^m + \cdots \tag{2.4}$$

where $G = G(\xi)$ satisfies the second order LODE in the form

$$G'' + \lambda G' + \mu G = 0, \tag{2.5}$$

$\alpha_m, \ldots$, λ and μ are constants to be determined later, $\alpha_m \neq 0$, the unwritten part in (2.4) is also a polynomial in $(\frac{G'}{G})$, but the degree of which is generally equal to or less than $m - 1$, the positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in ODE (2.3).

Step 3. By substituting (2.4) into Eq. (2.3) and using the second order LODE (2.5), collecting all terms with the same order of $(\frac{G'}{G})$ together, the left-hand side of Eq. (2.3) is converted into another polynomial in $(\frac{G'}{G})$. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $\alpha_m, \ldots$, λ and μ.

Step 4. Assuming that the constants $\alpha_m, \ldots$, λ and μ can be obtained by solving the algebraic equations in Step 3, since the general solutions of the second order LODE (2.5) have been well known for us, then substituting $\alpha_m, \ldots$ and the general solutions of Eq. (2.5) into (2.4) we have more travelling wave solutions of the nonlinear evolution Eq. (2.1).

In the subsequent sections we will illustrate the proposed method in detail with various nonlinear evolution equations in mathematical physics.

3. Integrable sixth-order Drinfeld–Sokolov–Satsuma–Hirota equation

We begin with the integrable sixth-order Drinfeld–Sokolov–Satsuma–Hirota equation:

$$w_t - 6ww_x + w_{xxx} - 6v_x = 0, \tag{3.1}$$

$$v_t - 2v_{xxx} + 6wv_x = 0, \tag{3.2}$$

In order to obtain the travelling wave solutions of (3.1) and (3.2), we suppose that

$$w(x,t) = w(\xi), \quad v(x,t) = v(\xi), \quad \xi = x - ct, \tag{3.3}$$

c is a constant that to be determined later.