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A new filled function algorithm for constrained global optimization problems

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ABSTRACT

A new filled function with one parameter is proposed for solving constrained global optimization problems without the coercive condition, in which the filled function contains neither exponential term nor fractional term and is easy to be calculated. A corresponding filled function algorithm is established based on analysis of the properties of the filled function. At last, we perform numerical experiments on some typical test problems using the algorithm and the detailed numerical results show that the algorithm is effective.

1. Introduction

In recent years, many new theoretical and computational contributions have been developed for solving global optimization problems. The filled function method is one of effective and practical global algorithms. The filled function method was firstly proposed by Ge and Qin in [1], which was used to solve the global minimizer of unconstrained multi-extremum function. The key part of the filled function method is the filled function, which is applied to find another smaller local minimum point of the objective function from the current local minimum point. The filled function method mainly consists of two phases, i.e., local minimization phase and filling phase, and the two phases are carried out alternately until a global minimizer is found.

The tunneling method presented by Levy and Montalvo in [2], is essentially similar to the filled function method. The tunneling method is also composed of two phases, i.e., local minimization phase and tunneling phase. The two phases are used alternately to search for the better minimizer. That is, the filled function method and the tunneling method are common in phase I. The only difference between a filled function and a tunneling function is the way used to find another better local minimizer of the objective function in phase II.

Since Ge and Qin presented the filled function method, rapid progress has been made both in theories and practical applications for the filled function method. For example, papers [1–5] were devoted to unconstrained global optimization problems and papers [6,7] studied box constrained global optimization problems. Due to most of the optimization problems have the general constrained condition, how to solve the global optimization problems with general constraints is significant in practice. There has been much recent research on the filled function algorithms for these problems, such as papers [8–16]. Although these algorithms have their own advantages, they have some defects in some degree, such as the constructed filled functions either needing more than one adjustable parameter or including exponential or fractional term, which result in numerical complexity during computation. We draw inspiration from papers [12,15] and construct a new simple filled function, which has only one adjustable parameter, contains neither exponential term nor fractional term and needs not the

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coercive assumption in this paper. Hence the minimization of the filled function will be easy to perform when a practical constrained optimization problem is being solved.

The rest of this paper is organized as follows. Following this introduction, in Section 2, we will present the basic knowledge that is basic for the subsequent sections. In Section 3, a new filled function will be proposed and its properties will be discussed in detail. Furthermore, we establish a corresponding filled function algorithm and numerical results for some typical test problems are reported to indicate feasibility and efficiency of the new algorithm in Section 4. In the last section, some conclusions are drawn.

2. Basic knowledge

In this paper, we consider the following constrained global optimization problem:

$$\min_{x\in S} f(x), \tag{P}$$

where $S = \{x \in X | g_i(x) \leqslant 0, i \in \Omega\}$, $X \subset R^n$ is a bounded box set, the functions $f(x)$, $g_i(x)$, $i \in \Omega : X \to R$ are continuously differentiable in R^n, and $\Omega = \{1, 2, \ldots, m\}$ is index set.

For convenience, we denote the set of local minimizers for problem (P) by $L(P)$ and the set of the global minimizers for problem (P) by $G(P)$. Let $\text{int}S = \{x \in X | g_i(x) < 0, i \in \Omega\}$, where $\text{int}S$ denotes the interior of set S.

To begin with, we make the following assumptions:

Assumption 2.1. The number of minimizers of problem (P) may be infinite, but the number of different function values at the minimizers is finite.

Assumption 2.2. The set S satisfies that $cl\ intS = clS$ and $\text{int}S$ is not empty, where clS denotes the closure of set S.

Assumption 2.3. Assume that x^* is a local minimizer of problem (P).

By Assumption 2.2, we know that for any $x_0 \in S$, there exists a sequence $\{x_n\} \subset \text{int}S$, such that $\lim_{n\to\infty} x_n = x_0$.

Now, we present the definition of the filled function for the constrained optimization problem in paper [9], which will be used in this paper.

Definition 2.1. A function $H(x, x^*, A)$ is said to be a filled function of problem (P) at a local minimizer x^* in which A is an adjustable parameter, if it satisfies the following conditions:

(1) x^* is a strictly local maximizer of $H(x, x^*, A)$ on X.
(2) For any $x \in S_1 \setminus \{x^*\}$ or $x \in X \setminus S$, $\nabla H(x, x^*, A) \neq 0$, where $S_1 = \{x \in S | f(x) \geqslant f(x^*)\}$.
(3) If $S_2 = \{x \in S | f(x) < f(x^*)\}$ is not empty, then there exists a point $x_1^* \in S_2$ such that x_1^* is a local minimizer of $H(x, x^*, A)$.

The significance of Definition 2.1 for the filled function is in that if x^* is a local minimizer but not a global minimizer of problem (P), the local minimizer x_0^* of the filled function must be a feasible point and be a much better minimizer than x^* for problem (P). In fact, from conditions (1) and (3), the local minimizer x_0^* of the filled function can be obtained by using any algorithm for unconstrained optimization problems beginning from any initial point in the neighborhood of x^*. The condition (2) shows that the point x_0^* must be in the feasible region S and be a better local minimizer than the current local minimizer x^* for problem (P).

3. A new filled function and its properties

Based on the basic knowledge, we establish a new filled function with one parameter for problem (P) as follows

$$p(x, x^*, A) = -A[f(x) - f(x^*)]^2 - \ln(1 + \|x - x^*\|^2) + A \min[0, \max(f(x) - f(x^*), g_i(x), i \in \Omega)],$$

where $A > 0$ is an adjustable parameter and x^* is a current local minimizer of $f(x)$ in S and $\|\cdot\|$ indicates the Euclidean vector norm.

The following theorems show that $p(x, x^*, A)$ is a filled function when $A > 0$ is big enough.

Theorem 3.1. *Assume that $x^* \in L(P)$, then x^* is a strictly local maximizer of $p(x, x^*, A)$.*

Proof. Obviously, we should prove that for all $x \in N(x^*, \delta)$ and $x \neq x^*$, it holds

$$p(x, x^*, A) < p(x^*, x^*, A),$$

where $\delta > 0$, $N(x^*, \delta)$ is a neighborhood of x^*.