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Numerical solution of the momentum and heat transfer equations for a hydromagnetic flow due to a stretching sheet of a non-uniform property micropolar liquid

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ABSTRACT

A study of the hydromagnetic flow due to a stretching sheet and heat transfer in an incompressible micropolar liquid is made. Temperature-dependent thermal conductivity and a non-uniform heat source/sink render the problem analytically intractable and hence a numerical study is made using the shooting method based on Runge–Kutta and Newton–Raphson methods. The two problems of horizontal and vertical stretching are considered to implement the numerical method. The former problem involves one-way coupling between linear momentum and heat transport equations and the latter involves two-way coupling. Further, both the problems involve two-way coupling between the non-linear equations of conservation of linear and angular momentums. A similarity transformation arrived at for the problem using the Lie group method facilitates the reduction of coupled, non-linear partial differential equations into coupled, non-linear ordinary differential equations. The algorithm for solving the resulting coupled, two-point, non-linear boundary value problem is presented in great detail in the paper. Extensive computation on velocity and temperature profiles is presented for a wide range of values of the parameters, for prescribed surface temperature (PST) and prescribed heat flux (PHF) boundary conditions.

1. Introduction

The problems in the non-linear regime of free or forced or mixed convection involving micropolar liquids are quite formidable to solve analytically or numerically. The reason for the challenging nature of these problems is the two-way coupling between the non-linear equations of conservation of linear and angular momentums. Further, there is a one-way coupling or two-way coupling between the conservations of linear momentum and energy depending on whether gravity effects are unimportant or important. The stretching sheet problem involving a micropolar liquid is one of those problems in this class of formidable problems. The problem is a prototype of many practical problems like polymer-extrusion processes and such others (see [40,1–3] and the references therein). In the problem a micropolar liquid is representative of the cooling liquid surrounding the stretching sheet that serves the purpose of a controlled cooling system in the presence of a magnetic field. The carrier liquid with the micron-sized suspended particles circumscribes to a micropolar description (see [15,16]). This liquid description is non-Newtonian in the sense that the stress tensor is asymmetric. The suspended

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particles assist the carrier liquid in quenching the heat in the sheet in such a way that the sheet has a desirable property required for the application in which it is sought. The suspended particles are electrically inert but the carrier liquid is assumed to have weak electrical conductivity. In the presence of a magnetic field, the distribution of the suspended particles is unaffected where as the carrier liquid is significantly influenced. The magnetohydrodynamic formulation of a problem is done in the following two ways:

(i) Chandrasekhar formulation or
(ii) Hartmann formulation

The Chandrasekhar formulation (see [10]) is used when the liquid has finite electrical conductivity and in this case one will have to take all of the Maxwell equations involving a magnetic field. Generally the appropriate Maxwell equations are in this case combined into a single equation known as the magnetic induction equation. The Hartmann formulation is used when the liquid has weak electrical conductivity and in this case the Lorentz force gives rise to a linear drag similar to Darcy friction seen in porous media (see [40,2,3]).

In view of the importance of the stretching sheet problem involving a micropolar liquid several investigators have focused their attention on this problem [5–9,11–14,17–35,37,38,43]. The above investigators have considered the horizontal stretching sheet problem in a uniform property micropolar liquid. It is the intension of the paper to examine and present the implementation of the shooting method in a problem that involves coupling, non-linearity, variable coefficients and also boundary conditions of the Dirichlet's and third-type. The intrinsic difficulty in handling a complex problem like this by the shooting method using a scientific procedure to estimate the missing initial values is presented in a great detail in the paper. The foremost objective of this paper is to lay bare all the nuances of the shooting method to help the reader get used to the method and apply the same to his/her problems with least difficulty. In our considered opinion the horizontal and vertical stretching sheet problems involving micropolar liquids are the best serving examples for demonstration of the shooting method. Work is in progress amongst our research group to apply the shooting method to eigen boundary value problems involving a micropolar liquid.

In the next two sections we take up the mathematical formulation of the horizontal and vertical stretching sheet problems.

2. Mathematical formulation for a horizontal stretching sheet

We consider a steady, two-dimensional, coerced flow due to an impermeable stretching sheet, of an incompressible, electrically conducting, micropolar liquid. The flow is generated by the action of two equal and opposite forces along the x-axis with the y-axis being normal to the flow. The sheet is stretched with a velocity $u_w(x)$ which is proportional to the distance from the origin (Fig. 1a) and it is assumed to be warmer than the ambient liquid, i.e., $t_w(x) > t_\infty$. The flow field is further

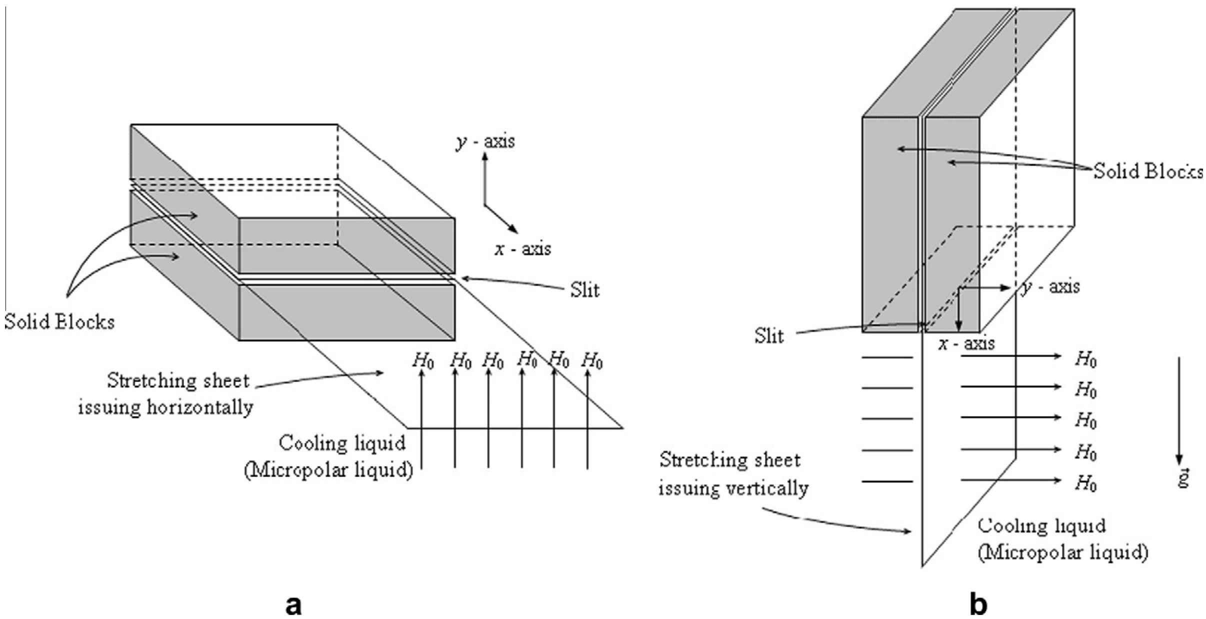

Fig. 1. Schematic of a polymer extrusion process in absence/presence of gravity effects.