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Finite-time boundedness and L_2-gain analysis for switched delay systems with norm-bounded disturbance

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ABSTRACT

Finite-time boundedness and finite-time weighted L_2-gain for a class of switched delay systems with time-varying exogenous disturbances are studied. Based on the average dwell-time technique, sufficient conditions which guarantee the switched linear system with time-delay is finite-time bounded and has finite-time weighted L_2-gain are given. These conditions are delay-dependent and are given in terms of linear matrix inequalities. Detail proofs are given by using multiple Lyapunov-like functions. An example is employed to verify the efficiency of the proposed method.

1. Introduction

Switched systems, which consist of a family of subsystems described by differential or difference equations and a switching law that orchestrates switching between these subsystems, belong to a special class of hybrid systems. Recently, switched systems have received a great deal of attention, such as stability [1–10] and controllability and observability [11–13]. This is due to the fact that switched systems have numerous applications in mechanical control systems, automotive industry, traffic control, switching power converters, and many other fields.

Time-delay is a common phenomenon and is unavoidable in engineering control design. Switched systems with time-delay have strong engineering background, such as power systems [14,15] and networked control systems [16,17]. However, due to the interaction among continuous-time dynamics, discrete-time dynamics and time-delay, the dynamics of switched systems with time-delay becomes more complex than switched systems without time-delay and time-delay systems without switching. Therefore, the study of switched systems with time-delay is very interesting and challenging. Recently, based on multiple Lyapunov-like functions method, many valuable results on such systems have been developed. In [18], by using the average dwell-time technique, Lyapunov stability and L_2-gain were analyzed for a class of switched systems with time-varying delay. In [19], for a class of uncertain discrete-time switched systems with mode-dependent time delays, robust stability analysis and H_∞ control problem were discussed. In [20], by designing a class of state-based switching signals, the problem of stabilization for switched linear systems with mode-dependent time-varying delays was solved.

Up to now, most of existing literature related to stability of switched systems focuses on Lyapunov asymptotic stability, which is defined over an infinite time interval. However, in practice, one is interested in not only system stability (usually in the sense of Lyapunov) but also a bound of system trajectories over a fixed short time, such as networked control systems [21,22] and network congestion control [23]. In addition, a system could be Lyapunov stable but completely useless because it possesses undesirable transient performances, such as the system with saturation elements in the control loop. To study the transient performances of a system, the concept of short time stability, i.e., finite-time stability, was introduced in [25].

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Specifically, a system is said to be finite-time stable if, given a bound on the initial condition, its state remains within a prescribed bound in a fixed time interval. Note that finite-time stability and Lyapunov asymptotic stability are independent concepts: a system could be finite-time stable but not Lyapunov asymptotically stable, and vice versa [27]. In addition, it should be emphasized that practical stability also studies the boundedness of system trajectory. Practical stability means that the corresponding controller can drive the system to an arbitrarily neighborhood around the origin, which is also defined over an infinite time interval.

Some early results on finite-time stability can be found in [24–26]. Recently, based on linear matrix inequality theory, many valuable results have been obtained for this type of stability [26–35]. In [27–29], the authors introduced the concept of finite-time boundedness which is an extension of finite-time stability, and presented some sufficient conditions for finite-time boundedness and stabilization of continuous-time systems or discrete-time systems. In [30], finite-time stabilization of linear time-varying systems has been studied. In [31–33], finite-time control problem for the impulsive systems was discussed. In [34], for a class of nonlinear quadratic systems, sufficient conditions for finite-time stability and stabilization of were also presented. For more analysis and synthesis results of finite control problem, the readers are referred to the literature [35,36] and the references therein. In addition, it should be pointed out that the authors of [37–40] have presented some results of finite-time stability for different systems, but finite-time stability in those systems which implies Lyapunov stability and finite-time convergence is different from that in this paper and [23–35].

So far, Lyapunov stability analysis for switched systems with time delay and finite-time stability for different systems have been extensively studied by many researchers. However, to the best of authors' knowledge, there is no result available yet on finite-time stability of switched systems with time-delay. For the switched systems without time-delay, in [41], practical stability and finite-time stability were discussed. For ease of computation, in [42], based on linear matrix inequalities, finite-time stability and stabilization conditions were developed. Considering the wide application of switched systems with time-delay and the requirement for transient behavior in engineering fields, it motivates us to investigate finite-time stability and finite-time boundedness for a class of switched linear systems with time-delay. Our contributions are given as follows: (1) Definitions of finite-time boundedness and finite-time weighted L_2-gain are extended to switched linear systems with time-delay. The system under consideration is subject to time-varying norm-bounded exogenous disturbance. (2) Sufficient conditions for finite-time boundedness and finite-time weighted L_2-gain of switched linear systems with time-delay are given.

The paper is organized as follows. In Section 2, some notations and problem formulations are presented. In Section 3, based on linear matrix inequalities, sufficient conditions which guarantee finite-time boundedness of switched linear systems with time-delay are given. Sufficient conditions which guarantee that system has finite-time weighted L_2-gain are presented in Section 4. Finally, an example is presented to illustrate the efficiency of the proposed method in Section 5. Concluding remarks are given in Section 6.

2. Preliminaries and problem formulation

In this paper, let $P > 0$ ($P \geqslant 0$, $P < 0$, $P \leqslant 0$) denote a symmetric positive definite (positive-semidefinite, negative definite, negative-semidefinite) matrix P. For any symmetric matrix P, $\lambda_{max}(P)$ and $\lambda_{min}(P)$ denote the maximum and minimum eigenvalues of matrix P, respectively. The identity matrix of order n is denoted as I_n (or, simply, I if no confusion arises).

Consider a switched linear systems with time-delay as follows

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}x(t-h) + G_{\sigma(t)}\omega(t), \\ z(t) = C_{\sigma(t)}x(t) + D_{\sigma(t)}\omega(t), \quad t \geqslant 0, \\ x(t) = \varphi(t), \quad t \in [-h, 0], \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state, $z(t) \in R^m$ is the control output, $A_{\sigma(t)}$, $B_{\sigma(t)}$, $C_{\sigma(t)}$ and $D_{\sigma(t)}$ are constant real matrices, $\varphi(t)$ is a differentiable vector-valued initial function on $[-h,0]$, $h > 0$ denotes the constant delay, $\omega(t)$ is time-varying exogenous disturbance and satisfies Assumption 2, $\sigma(t) : [0,\infty) \to M = \{1,2,\ldots,m\}$ is the switching signal which is a piecewise constant function depending on time t or state $x(t)$, and m is the number of subsystems.

Corresponding to the switching signal $\sigma(t)$, we have the following switching sequence:

$$\{x_0; (i_0, t_0), \ldots, (i_k, t_k), \ldots, | i_k \in M,\ k = 0, 1, \ldots\},$$

in which t_0 is the initial time, x_0 is the initial state and the i_kth subsystem is activated when $t \in [t_k, t_{k+1})$.

Assumption 1. The state of switched linear system does not jump at switching instants, i.e., the trajectory $x(t)$ is everywhere continuous. Switching signal $\sigma(t)$ has finite switching number in any finite interval time.

Assumption 2. The external disturbances $\omega(t)$ is time-varying and satisfies the constraint $\int_0^\infty \omega^T(t)\omega(t)dt \leqslant d,\ d \geqslant 0$.

It should be pointed out that the assumption about the external disturbances $\omega(t)$ in this paper is different from that of [27,31,32], where the external disturbances is constant.