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Radiation of water waves by a submerged nearly circular plate

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ABSTRACT

A thin nearly circular plate is submerged below the free surface of deep water. The problem is reduced to a hypersingular integral equation over the surface of the plate which is conformally mapped onto the unit disc. The solution is computed by a spectral method proved to be efficient for the case of a circular disc. Numerical results are presented for the heave added mass and damping coefficients for two types of nearly circular plates.

1. Introduction

A horizontally submerged thin plate is a component present in several structures used in offshore and coastal engineering. One of the major innovations used in the renewable energy sector is semi-submersible floating offshore wind turbines (FOWT). These structures often have column-stabilised platforms with submerged plates which provide extra heave added mass, wave damping far removed from wave excitation [1,2].

A number of authors have considered the radiation or scattering by a submerged thin plate. Yu and Chwang [3] have used matched eigenfunctions expansions for studying the scattering by a horizontal disc in water of finite depth. Not long after, the wave scattering and radiation by a submerged elliptical disc were investigated by Zhang and Williams [4,5] by similar techniques.

Martin and Farina [6] have considered the radiation of water waves by a submerged horizontal solid disc. They transformed the governing hypersingular integral equation for the jump in the velocity potential across the plate, $[\phi]$ into a one-dimensional Fredholm integral equation of the second kind for a new unknown function; the new equation is a generalisation of Love's integral equation, common in the theory of electrostatics of a circular-plate capacitor [7]. Numerical results of the heaving added mass and damping were presented. Farina [8] extended this work by considering the effects of taking the disc very close to the free surface and relating the hydrodynamic force to resonant frequencies. Both numerical and asymptotic methods have been used. The three-dimensional scattering by a thin disc, in deep water was investigated by Farina and Martin [9]. The authors solved the governing hypersingular integral equation numerically using a

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expansion-collocation spectral method. Similarly to the radiation problem, they found that the scattering problem presents a strong dependence on the frequency when the plate is close to the free surface. Relationships between the scattering cross-section and the peaks in the added mass have been presented.

Yu [10] uses analytical, numerical, and semi-empirical methods and summarises the functional performance of a submerged and essentially horizontal plate for offshore wave control. The authors focus on the hydrodynamics force and on the reflection and transmission coefficients.

The wave radiation by disc which is perturbed out of its original plane was considered by Ziebell and Farina [11]. Perturbing the plate surface, the authors formulated the problem in terms of hypersingular integral equations and the hydrodynamic force on circular caps and random rough discs were computed.

Recently, Porter [12] used a method based on Fourier/Hankel transforms to treat the scattering and the radiation by thin horizontal plates in two dimensions and also by a circular disc in three dimensions. For the latter, numerical results for the scattering cross section were presented and compared with [9].

In this work we consider the radiation of water waves by nearly circular plates. We formulate the problem by means of a hypersingular integral equation and as in the work by Martin on flat pressurised cracks [13], the new geometries are conformally mapped onto the unit disc. This preserves the kernel singularity and an efficient spectral method based on orthogonal polynomials can thus be applied to obtain the numerical solution. The convergence of the Galerkin version of this method has been recently proved by Farina et al. [14].

The heave added mass and damping coefficients are computed and numerical results are shown for two different nearly circular plates.

2. Formulation

A Cartesian coordinate system is chosen, in which z is directed vertically downwards into the fluid. We take the mean free surface lying at $z = 0$. We assume the presence of a submerged body into the fluid with a smooth, closed and bounded surface S. We suppose that the motions of the fluid are of small-amplitude, time-harmonic, that the fluid is incompressible and inviscid, and that the motion is irrotational. We denote ϕ as the potential flow and $[\phi]$ as the discontinuity in ϕ across S. Thus, the time-dependent velocity potential is $\mathrm{Re}\{\phi(x, z, t)\}\mathrm{e}^{-i\omega t}$, where ω is the angular frequency.

The conditions to be satisfied by ϕ are Laplace's equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\phi = 0 \quad \text{in the fluid,}$$

along with the free-surface condition

$$K\phi + \frac{\partial \phi}{\partial z} = 0 \quad \text{on} \quad z = 0,$$

where $K = \omega^2/g$; g being the acceleration due to gravity.

On the surface of the body, the normal velocity is prescribed by

$$\frac{\partial \phi}{\partial n} = V \quad \text{on} \quad S, \tag{1}$$

where V is a given function and $\frac{\partial}{\partial n}$ denotes the normal derivative on S.

Additionally, ϕ must satisfy a radiation condition:

$$r^{1/2}\left(\frac{\partial \phi}{\partial r} - iK\phi\right) \to 0 \quad \text{when } r = (x^2 + y^2)^{1/2} \to \infty.$$

In what follows, the points P, Q denote points in the fluid and the points p, q denote points on the submerged body.

The free surface Green function for this problem is given by

$$G(P, Q) \equiv G(x_0, y_0, z_0; x, y, z) = G_0(R, z - z_0) + G_1(R, z + z_0), \tag{2}$$

where $R = ((x - x_0)^2 + (y - y_0)^2)^{1/2}$, $G_0(R, z - z_0) = (R^2 + (z - z_0)^2)^{-1/2}$ and

$$G_1(R, z + z_0) = \int_0^{\infty} \mathrm{e}^{-k(z+z_0)} J_0(kR) \frac{k + K}{k - K}\, dk. \tag{3}$$

Here J_0 is the Bessel function of order zero. The integral above defining G_1 has its contour of integration passing below the singularity K to satisfy the radiation condition. G also satisfies the free surface condition, the Laplace equation, and has a weak singularity at $P = Q$.

For any harmonic function ϕ, satisfying $\phi = O(r^{-1})$ as $r \to \infty$, we have from Green's second identity, the following integral representation.

$$\phi(P) = \frac{1}{4\pi} \int_S \left(\phi(q) \frac{\partial}{\partial n_q} G(P, q) - G(P, q) \frac{\partial \phi}{\partial n_q}\right) dS_q, \tag{4}$$

where $\frac{\partial}{\partial n_q}$ denotes normal differentiation at q on S.