Journal of Computational and Applied Mathematics 289 (2015) 173–195

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

A probabilistic model for predicting the uncertainties of the humid stiction phenomenon on hard materials

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HIGHLIGHTS

- An alternative to the statistical method is proposed to evaluate the adhesive forces.
- Numerical surfaces are generated from a psd function defined from AFM measurements.
- An asperity approach is applied on these surfaces to compute the adhesive forces.
- The method is able to capture uncertainties and size effects in the adhesive forces.
- The method can be used to predict the stiction in MEMS devices.

ARTICLE INFO

Article history:
Received 20 September 2014
Received in revised form 11 December 2014

Keywords:
Stiction
Capillary force
Surface generation
Hertz model
DMT model
GW approach

ABSTRACT

Stiction is a major failure in microelectromechanical system (MEMS) devices in which two contacting surfaces can remain stuck together because of the adhesive forces. Due to the difference between the surfaces roughness and the adhesive force range, the real contact areas are usually smaller than the apparent one, resulting in a scatter in the adhesive forces. Consequently, the stiction is an uncertain phenomenon. In this work, we develop a probabilistic model to predict the uncertainties of stiction due to the capillary forces acting on stiff materials. This model contains two levels: at the deterministic level, the model can predict the pull-out adhesive contact forces for a given surface topology; at the probabilistic level, the model generates independent identically distributed surfaces on which the deterministic solution can be applied to evaluate the uncertainties related to the stiction phenomenon.

1. Introduction

Because of their intrinsic advantages, e.g. miniature sizes, low power requirement, and reduced manufacturing cost, many micro-electro-mechanical system (MEMS) devices, such as accelerometers, digital mirrors, pressure sensors, gyroscopes, and resonators, have been innovated and successfully applied in the industry. In spite of their advantages, due to the inherent characteristic of MEMS, such as the large surface area to volume ratio, the relative smoothness of surfaces, the

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http://dx.doi.org/10.1016/j.cam.2015.02.022

small interfacial gap, and the small restoring forces of the compliant structures, the contact between their components can lead to the permanent adhesion of their moving parts. This physical phenomenon, named "stiction", is a common failure of MEMS devices [1].

The stiction is the combination of stick and friction which is due to the adhesive forces such as van der Waals (vdW) forces, capillary forces *etc*. In humid conditions, due to the presence of condensing water between the two contacting surfaces, the capillary forces resulting from the negative relative pressure inside the condensing water layers can lead to the stiction of these surfaces. This phenomenon is named "humid stiction". The condensing water layers, the root of humid stiction, strongly depend on the surfaces topologies on which they are formed [1]. However, the topology of rough MEMS surfaces is characterized by a degree of randomness at the nano-scale, making the humid stiction an uncertain phenomenon. This uncertainty was experimentally observed: in [1], at a 70%-humidity level, the difference between the highest and the lowest adhesive energies – the energies to completely separate the contacting surfaces – was found to be about 200%; in [2], at a 85%-humidity level, this difference was about 300%. Therefore, probabilistic numerical contact models are required in order to make the humid stiction failure predictable and avoidable. To fulfill these requirements, these models should have two features: (i) at the deterministic level, for a well-defined contact surface topology, the numerical model should be able to predict the contact forces; (ii) at the uncertainty quantification level, the numerical contact model should be able to quantify the uncertainties on the contact forces and on the risk of stiction.

For this first feature, the numerical contact theories have a long history. Based on the Hertz non-adhesive elastic contact theory for spherical asperities [3], three common asperity contact theories were developed for the adhesive case, (i) the Johnson–Kendall–Robert (JKR) theory [4], (ii) the Derjaguin–Muller–Toporov (DMT) theory [5], and (iii) the Maugis theory [6,7]. The JKR theory assumes that the adhesive forces are applied solely inside the contact area. This theory can thus be applied for soft materials and short-range adhesive forces, such as vdW forces. In contrast, the DMT theory assumes that the adhesive contact forces are applied outside the contact area, consequently it is applicable for hard materials and long-range adhesive forces, such as capillary forces. The third theory, the Maugis one, is a transition solution between these two models. For the humid stiction case of interest, both the Maugis and DMT theories are applicable. Besides those asperity models, the adhesive contact, due to vdW forces, between asperities has also been studying using molecular dynamics (MD) methods [8]. MD methods were also used to study the adhesive interactions due to vdW forces of silicon [9], silica [10], and silicon carbide [11] nano-particles. These studies have shown that the JKR and DMT models are valid up to given contact interference distances. Finally, the adhesive interaction between a flat surface and a sphere has been modeled using the finite element method [12] by representing the vdW forces through a Lennard-Jones potential. This model was compared to MD simulations.

To evaluate the contact forces at the surface level from the asperity contact forces, the most common method is the statistical approach [13–15]. This so-called "Greenwood–Williamson" (GW) method was considered in several stiction models [1,16,17], including for structural finite element analyzes [18]. In this approach, two important assumptions are made: (i) the asperities are represented at their local maximum by their curvature; (ii) the interactions between the contacting asperities of a surface are neglected. By considering the surface as a second order stationary Gaussian random field, the distribution of maxima can be obtained from the statistical parameters of this field. Using the previous two assumptions, the contact forces can be evaluated at the surface level by integrating the asperity contact forces weighted by the statistical distribution of maxima.

However, there are some cautions to be exercised when applying this statistical approach. For instance, the validity of the first assumption is questionable in practice as one asperity can include many maxima. This limitation was reported by Greenwood [19], one of the authors of statistical approach. In addition, in the case of humid stiction, the second assumption, *i.e.* neglecting the interaction among the asperities of a surface, has a short validity range since the condensing water layers of different contacting asperities can merge together, especially at high humidity levels. This merging phenomenon, named "saturation", was reported in [1]. Moreover, because of the Gaussian nature of the random field, the surface contact forces obtained with the statistical approach can be seen as the average solution of contact forces from (an infinite number of) different individual surfaces. While the result of the statistical approach is an average solution corresponding to an infinite surface, the experimental results of humid stiction tests on finite surfaces show an important uncertainty [1,2]. In addition, with the statistical approach, because of the Gaussian nature of the random fields, asperities of much higher height than the ones experimentally observed are considered in the distribution. Although their probability of existence is small, their induced contact forces are much higher and the solution converges to values different from the ones observed experimentally by several orders of magnitude. In [1], to obtain numerical results of the same order of magnitude as the experimental ones, the authors have changed the integral limits of the statistical integration process. However the determination of these limits is delicate. Moreover, in the case of poly-silicon, the considered material in this paper, there is little plasticity and the determination of the limits cannot be physically motivated.

Besides the statistical approach, the finite element method is also a candidate to model the stiction of rough surfaces [20,21]. However a finite element model is costly in terms of computation due to the high ratio of the surface size to the height of the condensing water layers, and due to the non-linearity of the adhesive contact problem, which limits its applicability when performing uncertainty quantification.

In this work we propose an alternative to the usual GW asperity contact theory formulation.

At first, we develop a modified DMT model that is applied on a defined surface topology, either experimentally obtained or numerically generated. In this model the Hertz contact repulsive forces and the humid adhesive forces are computed