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A wavelet multi-scale method for the inverse problem of diffuse optical tomography

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ABSTRACT

This paper deals with the estimation of optical property distributions of participating media from a set of light sources and sensors located on the boundaries of the medium. This is the so-called diffuse optical tomography problem. Such a non-linear ill-posed inverse problem is solved through the minimization of a cost function which depends on the discrepancy, in a least-square sense, between some measurements and associated predictions. In the present case, predictions are based on the diffuse approximation model in the frequency domain while the optimization problem is solved by the L-BFGS algorithm. To cope with the local convergence property of the optimizer and the presence of numerous local minima in the cost function, a wavelet multi-scale method associated with the L-BFGS method is developed, implemented, and validated. This method relies on a reformulation of the original inverse problem into a sequence of sub-inverse problems of different scales using wavelet transform, from the largest scale to the smallest one. Numerical results show that the proposed method brings more stability with respect to the ordinary L-BFGS method and enhances the reconstructed images for most of initial guesses of optical properties.

1. Introduction

The diffuse optical tomography (DOT) problem is an inverse problem in which the spatial distributions of absorption and scattering coefficients of a participating medium are sought from a set of light sources and sensors located on the frontier of the probed medium [1]. The main application of the DOT is imaging biological tissues based on the fact that knowledge of the optical properties provides information on the physiopathological condition of these tissues. A thorough review of work on applications of the DOT in the biomedical domain can be found in [2].

Such a non-linear ill-posed inverse problem is solved through the minimization of a cost function which depends on the discrepancy, in a least-square sense, between some measurements and associated predictions. The methods employed to solve this problem include the non-linear conjugate-gradient method [3], Gauss–Newton based methods [4–7], the L-BFGS method [8–12], shape-based reconstruction method [13,14] or, in a Bayesian framework, the approximation error method [15,16]. Regarding the first three listed methods, some problems remain to be overcome such as the stability with respect to the initial guesses and the blurring effect of the reconstructed images due to the need for relatively strong regularization tools [17,14].

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A relatively new method has emerged in the field of inversion, namely the wavelet multi-scale method [18–25]. This method relies on a reformulation of the original inverse problem into a sequence of sub-inverse problems of different scales using wavelet transform, from the largest scale to the smallest one. Successful applications of this method include the inversion of the Maxwell equations [19], the identification of space-dependent porosity in fluid-saturated porous media [20,21], the inversion of the two-dimensional acoustic wave equation [22], the reconstruction of permittivity distribution by the inversion of the electrical capacitance tomography model [23], the parameter estimation of elliptical partial differential equations [18,24] and the identification of space-dependent permeability in a nonlinear diffusion equation [25]. It is shown in these papers that the wavelet multi-scale method can enhance stability of inversion, accelerate convergence and cope with the presence of local minima to reach the global minimum. The rationale behind this success is that the cost function shows stronger convexity and has less minima at larger scale such that the global minimum can be achieved. Best results can thus be obtained, using this solution to the initialization of the optimization problem, at the lower scale until finding the minimum at the original scale.

This paper is dedicated to the application of a wavelet multi-scale method associated with the L-BFGS method to the specific inverse problem of diffuse optical tomography (DOT) and aims to improve the efficiency of the ordinary L-BFGS algorithm. To the best knowledge of the authors, this is the first time that the L-BFGS algorithm is coupled with the multi-scale method in solving the DOT inverse problem.

The paper is organized as follows: Section 2 shortly summarizes the models used in the area of optical tomography, namely the radiative transfer equation (RTE) and the related diffuse approximation (DA) model; Section 3 introduces the cost function, provides the expression of the continuous cost function gradient and defines the optimization problem; Section 4 describes the L-BFGS algorithm; Section 5 presents an introduction to the wavelet theory and the proposed wavelet multi-scale method; and Section 6 provides selected numerical results to assess the proposed method.

But before presenting the work carried-out in the paper, some clarification is mandatory to fully understand the originality of the work. Indeed, although some papers found in the literature deal with the DOT problem using the wavelet tool such as [26–29], these works show significant differences with the current research:

- In [26], the wavelet transform is used to denoise and compress noisy experimental data before performing the reconstruction. In this paper, the use of the wavelet transform is not limited to the data filtering, the wavelet transform is used in all stages of the inverse problem.
- In [27–29], the DOT problem is solved through the solution of a linear perturbation equation in which the main ingredient is a matrix of weights that are essentially the derivatives of the detector readings with respect to the optical coefficients in the reference medium. Several iterative algorithms have been used to solve this matrix system, including the conjugate gradient descent [27] and the total least squares [28,29]. In all these algorithms, the multi-scale approach consists in multiplying both terms of the matrix system by wavelet transform matrices at each scale to obtain solutions at the different scales. In the present paper, the methodology is totally different: the DOT problem is solved through the minimization of a least squares cost function by the L-BFGS algorithm for which the main ingredient is the cost function gradient. Thus, the multi-scale approach is mainly based on the representation of this gradient at the different scales.
- In [27–29], the steady-state diffuse approximation (DA) was considered while the DA in the frequency domain is used in the present case. As a result, only the absorption coefficient was reconstructed in [27–29] assuming the reduced scattering coefficient known within the media, while the simultaneous reconstruction of both the absorption and reduced scattering coefficients is performed in this paper, which represents an additional difficulty.

As a final comment about the current paper, it is worth mentioning that a wavelet multi-scale method associated with the L-BFGS algorithm is compared to the ordinary L-BFGS method. Comparisons with other methods found in the literature such as the Gauss–Newton method is out of the scope of the paper. However, let us point-out that the L-BFGS method has proven its worth in the field of the optical tomography as shown by the following papers [8–11].

2. The diffuse optical tomography problem

In the framework of radiative heat transfer in participating medium, the quantity of interest is the radiative intensity I. The equation describing the spectral radiative intensity distribution in space and time at the frequency ν, I_ν, in the medium is the RTE [30,31]:

$$\left(\frac{n}{c_0}\frac{\partial}{\partial t} + \mathbf{s}\cdot\nabla\right) I_\nu(\mathbf{r},\mathbf{s},t) = -(\kappa(\mathbf{r}) + \sigma(\mathbf{r}))I_\nu(\mathbf{r},\mathbf{s},t) + \sigma(\mathbf{r})\int_{\mathcal{S}^2} I_\nu(\mathbf{r},\mathbf{s}',t)\Psi(\mathbf{s},\mathbf{s}')\,d\mathbf{s}' + S_\nu(\mathbf{r},\mathbf{s},t) \quad \forall \mathbf{r}\in\Omega \tag{1}$$

where $I_\nu : \left(\Omega\times\mathcal{S}^2\times\mathbb{R}\right)\mapsto\mathbb{R}$, t is the time variable, $\mathbf{r} = (x, y, z)$ is the space variable, $\mathbf{s}$ is the direction, n is the refractive index (constant), c_0 is the speed of light in vacuum, $\kappa, \sigma : \Omega\mapsto\mathbb{R}^+$ are the absorption and scattering coefficients, respectively, Ψ is the scattering phase function, which describes the probability that a ray from the direction $\mathbf{s}'$ will be scattered into the direction $\mathbf{s}$, and S_ν represents volumetric source terms such as the spontaneous emission.

The RTE is an integro-differential equation that requires heavy computation to get accurate solutions. Thus, an estimation of optical property maps based on this forward model leads to a very cumbersome inverse problem to solve. The DOT