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First order partial differential equations with time delay and retardation of a state variable

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Abstract

We construct a finite difference scheme for the numerical solution of a first order partial differential equation with a time delay and retardation of a state variable. Such equations are used to model the dynamics of structured cell populations when age and maturity level are taken into account. For the supplied difference schemes the order of approximation, stability and convergence order are studied. We illustrate the obtained results with a test example.

Keywords: difference scheme, partial delay differential equation, retardation, time delay
2010 MSC: 65Q20, 65M06

1. Introduction

First order partial differential equations with time delay and retardation of a state variable, also known as advection equations with distributed parameters, arise in the modeling of dynamics of populations structured with respect to the cell size, the age of specimen, maturation level etc. [5, 12, 11]. The authors in [12] note that the dynamics are not only dependent on the behavior of the cell population numbers some time in the past (time delayed effects), but also that the population behavior at a given maturation level is dependent on the behavior at a previous maturation level (nonlocal effects). Thus, this important biological problem leads in a rather natural fashion to a complex mathematical problem involving delayed nonlocal dynamics described by a nonlinear advection equation. When diffusion is more dominant, such as in elasto-plasticity and in the theory of reactive contaminant transport, time delay can also occur and can be modelled through a convolution term, see e.g. [3, 4].

The qualitative theory of partial functional differential equations (PFDE) in general form is elaborated quite well (see, for example, [28] and references therein). Papers which deal with an advection equation with time delay and retardation of a state variable and its applications in cell dynamics usual consider questions of existence, uniqueness and global stability. As a general rule the equation is rewritten as a linear evolution problem in a Banach space and

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