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A model for non-isothermal flow and mineral precipitation and dissolution in a thin strip

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ABSTRACT

Motivated by porosity changes due to chemical reactions caused by injection of cold water in a geothermal reservoir, we propose a two-dimensional pore scale model of a thin strip. The pore scale model includes fluid flow, heat transport and reactive transport where changes in aperture is taken into account. The thin strip consists of void space and grains, where ions are transported in the fluid in the void space. At the interface between void and grain, ions are allowed to precipitate and become part of the grain, or conversely, minerals in the grain can dissolve and become part of the fluid flow, and we honor the possible change in aperture these two processes cause. We include temperature dependence and possible effects of the temperature in both fluid properties and in the mineral precipitation and dissolution reactions. For the pore scale model equations, we investigate the limit as the width of the thin strip approaches zero, deriving upscaled one-dimensional effective equations.

1. Introduction

Geochemistry has a substantial impact in exploiting geothermal systems. In a geothermal reservoir, the injected water and the in situ brine have different temperatures and chemical compositions. Also, the fluids flow through highly heterogeneous regions. Due to the varying chemical properties of the rocks, the temperature and the flow regimes can change significantly. As a consequence of flow and geochemical reactions, composition of reservoir fluids as well as reservoir rock properties will develop dynamically with time. Minerals dissolving and precipitating onto the reservoir matrix, can change the porosity and hence the permeability of the system substantially. Mineral solubility can change by the cooling of the rock, or by the different ion content in the in situ brine and in the injected water. The interaction between altering temperature, solute transport with mineral dissolution and precipitation and fluid flow is highly coupled and challenging to model appropriately as the relevant physical processes jointly affect each other [1]. The effect of changing porosity through the production period of the geothermal reservoir may have severe impact on operating conditions, as pores may close and block flow paths, or new pores may open to create enhanced flow conditions.

Injection of cold water into a geothermal reservoir can trigger the chemical reactions. The ion content of the injected water is normally different from the original groundwater, affecting the equilibrium state of the chemical system. Also,

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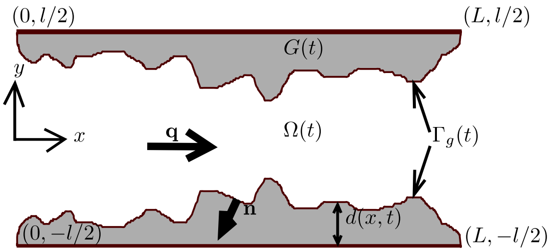

Fig. 1. Model of thin strip.

the solubility of several minerals are temperature dependent, hence the cooling of the porous medium can itself trigger chemical reactions. As reported from field studies and simulations, porosity and permeability changes due to precipitation and dissolution of minerals such as silica, quartz, anhydrite, gypsum and calcite have been observed [2–7]. Modeling of the mineral precipitation and dissolution is important in order to understand the processes and to better estimate to which extent the chemical reactions can affect the permeability of the porous medium.

When dealing with porosity changes, what happens at the pore scale is highly relevant. The pore geometry affects the reaction rates for the dissolution and precipitation process as the reactive surface area is changed, and the resulting permeability is affected by the pore geometry. To achieve expressions for both reaction rates and permeability that depend on the pore scale effects we start with a model at the pore scale, and derive the Darcy scale model by homogenization. We propose in this paper a pore scale model to investigate these matters. Pore scale models incorporating mineral precipitation and dissolution have been studied earlier in [8,9] and the corresponding Darcy scale models have been investigated further in [10,11]. These papers assume that the pore geometry is not changed by the chemical reactions, which is a valid assumption when the deposited or dissolved mineral layer is thin enough. Investigations honoring the porosity changes may be found in [12–14], where mineral precipitation and dissolution have been considered on either circular grains or in a thin strip. In these papers, the position of the interface between grain and void space is tracked, giving a problem with a free boundary. Similar models can also be obtained for biofilm growth [15], for drug release from collagen matrices [16], and on an evolving microstructure [17]. Recently, Kumar et al. [18] considered how to do numerical computations of a mineral precipitation and dissolution process in a thin strip with a free boundary.

In the spirit of [14], we consider mineral precipitation and dissolution in a thin strip and take into account the effect of temperature on the chemical reactions and on the fluid flow, giving a larger system of equations. Temperature changes can initiate or accelerate the rate of chemical reactions due to changes in solubility of the minerals. Also, the fluid flow is affected by the temperature changes due to changes in the fluid density and viscosity. For geothermal systems, the temperature dependence can be of high importance [19].

The structure of this paper is as follows. In Section 2 we discuss the pore scale model and the model equations describing the relevant processes on the pore scale. In Section 3 we perform formal homogenization on the model equations, obtaining upscaled equations valid when the width of the strip approaches zero. The paper ends with some concluding remarks on the resulting equations in Section 4.

2. Pore scale model

The pore model is represented by a two-dimensional thin strip with width l and length L, where L is much larger than l, and can be seen in Fig. 1.

We assume symmetry around the x-axis, hence the upper half of the strip is a reflection of the lower half. The width of the mineral part is $d(x, t)$ where $0 \leq d(x, t) < l/2$, as a greater width would clog the pore channel. The total domain Υ is the rectangle seen in the figure given by

$$\Upsilon = \{(x, y) \in R^2 | 0 \leq x \leq L, -l/2 \leq y \leq l/2\}.$$

The void space $\Omega(t)$ where fluid can flow is defined as

$$\Omega(t) = \{(x, y) \in R^2 | 0 \leq x \leq L, -(l/2 - d(x, t)) \leq y \leq (l/2 - d(x, t))\},$$

while the grain space $G(t)$ consisting of minerals is

$$G(t) = \{(x, y) \in R^2 | 0 \leq x \leq L, -l/2 \leq y \leq -(l/2 - d(x, t)) \vee (l/2 - d(x, t)) \leq y \leq l/2\}.$$

The void and grain spaces are separated by the interface $\Gamma_g(t)$ where mineral precipitation and dissolution can occur, and is given by

$$\Gamma_g(t) = \{(x, y) \in R^2 | 0 < x < L, y = \pm(l/2 - d(x, t))\}.$$

The inflow and outflow boundaries are

$$\Gamma_i(t) = \{(x, y) \in R^2 | x = 0, -(l/2 - d(x, t)) \leq y \leq (l/2 - d(x, t))\}$$