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Globally convergent and adaptive finite element methods in imaging of buried objects from experimental backscattering radar measurements

Larisa Beilina [a,*], Nguyen Trung Thành [b], Michael V. Klibanov [c], John Bondestam Malmberg [a]

[a] Department of Mathematical Sciences, Chalmers University of Technology and Gothenburg University, SE-42196 Gothenburg, Sweden

[b] Department of Mathematics, Iowa State University, Ames, IA, USA

[c] Department of Mathematics and Statistics University of North Carolina at Charlotte, Charlotte, NC 28223, USA

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ABSTRACT

We consider a two-stage numerical procedure for imaging of objects buried in dry sand using time-dependent backscattering experimental radar measurements. These measurements are generated by a single point source of electric pulses and are collected using a microwave scattering facility which was built at the University of North Carolina at Charlotte. Our imaging problem is formulated as the inverse problem of the reconstruction of the spatially distributed dielectric constant $\varepsilon_r(\mathbf{x})$, $\mathbf{x} \in \mathbb{R}^3$, which is an unknown coefficient in Maxwell's equations.

On the first stage the globally convergent method of Beilina and Klibanov (2012) is applied to get a good first approximation for the exact solution. Results of this stage were presented in Thành et al. (2014). On the second stage the locally convergent adaptive finite element method of Beilina (2011) is applied to refine the solution obtained on the first stage. The two-stage numerical procedure results in accurate imaging of all three components of interest of targets: shapes, locations and refractive indices. In this paper we briefly describe methods and present new reconstruction results for both stages.

1. Introduction

In this paper we consider the problem of reconstruction of refractive indices, shapes and locations of buried objects in the dry sand from backscattering time-dependent experimental data using the two-stage numerical procedure presented in [1–4]. Our problem is a coefficient inverse problem (CIP) for Maxwell's equations in three dimensions. Experimental data were collected using a microwave scattering facility which was built at the University of North Carolina at Charlotte, USA. Our experimental data were collected using a single location of the source. The backscattered signal was measured on a part of a plane. Our potential applications are in imaging of explosives, such as land mines and improvised explosive devices. This work is a continuation of our recent works on this topic, where we have treated a much simpler case of experimental data for targets placed in air [3,5,6].

* Corresponding author.
E-mail addresses: larisa@chalmers.se (L. Beilina), thanh@iastate.edu (N.T. Thành), mklibanv@uncc.edu (M.V. Klibanov), john.bondestam.malmberg@chalmers.se (J.B. Malmberg).

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The two-stage numerical procedure means that we combine two different methods to solve our CIP. On the first stage the globally convergent numerical method of [2] is applied in order to obtain a good first approximation for the exact solution without any a priori knowledge of a small neighborhood of this solution, see Section 2.9 of [2] as well as [7] for global convergence theorems. We presented results of reconstruction of the first stage in our publications [5,6] for objects placed in air. In our recent study [8] we presented reconstructions of twenty five (25) objects. This study has demonstrated that the method of [2] works well in estimating the dielectric constants (equivalently, refractive indices) and locations of buried objects.

It was proved in [9] that a minimizer of the Tikhonov functional is indeed closer to the exact solution than the first guess for this solution. Thus, it makes sense to apply the Tikhonov functional in order to refine the solution which we have obtained on the first stage of our two-stage numerical procedure. To do this, the locally convergent adaptive finite element method of [10] (adaptivity) is applied on the second stage. The adaptivity uses the solution of the first stage as the starting point in the minimization of a Tikhonov functional in order to obtain better approximations of refractive indices and shapes of objects on the adaptively refined meshes. It was shown in [3] that the adaptivity helps to accurately image simultaneously all three components of interest for targets placed in the air: refractive indices, shapes and locations.

Compared to the case of imaging of targets placed in air (see [3,5,6]), there are three main difficulties in imaging of buried targets: (i) the signals of targets are much weaker than those when the targets are in air, (ii) these signals may overlap with the reflection from the ground's surface, which makes them difficult to distinguish, and (iii) the reflection from the grounds surface may dominate the target's signals after the Laplace transform since the kernel of this transform decays exponentially with respect to time. We have handled this difficulty in [8] via a new data preprocessing procedure. This procedure results in preprocessed data, which are used as the input for our globally convergent algorithm, i.e. the input for the first stage of our method.

It is notable that we have experimentally observed a rare superresolution phenomenon and have numerically reconstructed the corresponding image, see Figs. 4(d) and 9. The resolution limit which follows from the Born approximation, i.e. the diffraction limit, is $\lambda/2$, where λ is the wavelength of the signal. In our experimental device $\lambda = 4.5$ centimeters (cm). We have resolved two targets at the distance of 1 cm $= \lambda/4.5$ between their surfaces. At the same time, the backscattering signal was measured at the distance of about 80 cm $\approx$ 18 wavelengths off the targets, i.e. in the far field zone. It was shown in, for instance [11], that the superresolution can occur because of nonlinear scattering, and our algorithm is nonlinear, including the step of extraction of the target's signal in our data preprocessing procedure [8]. Experimentally the superresolution phenomenon was demonstrated in [12]. We also refer to the recent work [13] where the superresolution is discussed.

An outline of this paper is as follows. In Section 2 we briefly describe the globally convergent method. In Section 3 we present the forward, inverse, and adjoint problems as well as the Tikhonov functional for the second stage. In Section 4 we describe the finite element method used in computations and in Section 5 we investigate general framework for a posteriori error estimation for CIPs. In Section 6 we describe the mesh refinement recommendation and the adaptive algorithm. In Section 7 we present results of our computations.

2. The first stage

In this section we state the forward and inverse problems which we consider on the first stage. We also briefly outline the globally convergent method of [2] and present the algorithm used in computations of the first stage.

2.1. Forward and inverse problems

Let $\Omega \subset \mathbb{R}^3$ be a convex bounded domain with the boundary $\partial\Omega \in C^3$. Denote the spatial coordinates by $\mathbf{x} = (x, y, z) \in \mathbb{R}^3$. Let $C^{k+\alpha}$ be Hölder spaces, where $k \geq 0$ is an integer and $\alpha \in (0, 1)$. We consider the propagation of the electromagnetic wave in $\mathbb{R}^3$ generated by an incident plane wave. On the first stage we model the wave propagation by the following Cauchy problem for the scalar wave equation

$$\varepsilon_{\mathrm{r}}(\mathbf{x}) \frac{\partial^2 u}{\partial t^2}(\mathbf{x}, t) - \Delta u(\mathbf{x}, t) = \delta(z - z_0) f(t), \quad (\mathbf{x}, t) \in \mathbb{R}^3 \times (0, \infty), \tag{1}$$

$$u(\mathbf{x}, 0) = 0, \qquad \frac{\partial u}{\partial t}(\mathbf{x}, 0) = 0, \quad \mathbf{x} \in \mathbb{R}^3. \tag{2}$$

Here $f(t) \not\equiv 0$ is the time-dependent waveform of the incident plane wave generated at the plane $\{z = z_0\}$ and propagating along the z-axis, and u is the total wave.

Let the function $E(\mathbf{x}, t) = (E_1, E_2, E_3)(\mathbf{x}, t)$ be the electric field. In our experiments the single non-zero component of the incident electric field is E_2 and we measure the backscattering function E_2, which is the voltage. Our mathematical model of the first stage uses only the single equation (1) with $u = E_2$ instead of the full Maxwell's system. Such approximation is reasonable, since it was shown numerically in [14] that the component E_2 of the electric field E dominates two other components in the case which we consider. Also, see [2] where a similar scalar wave equation was used to work with transmitted experimental data.

The function $\varepsilon_{\mathrm{r}}(\mathbf{x})$ in (1) represents the spatially distributed relative dielectric constant, i.e. the dielectric constant. It is known that $\varepsilon(\mathbf{x}) = \varepsilon_{\mathrm{r}}(\mathbf{x})\varepsilon_0$, where $\varepsilon(\mathbf{x})$ is the absolute dielectric permittivity of the material and ε_0 is the dielectric permittivity of vacuum. Both $\varepsilon(x)$ and ε_0 are measured in Farad/meter. Thus, $\varepsilon_{\mathrm{r}}(x)$ is dimensionless. We assume that ε_{r} is unknown