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On cost function transformations for the reduction of uncertain model parameters' impact towards the optimal solutions

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ABSTRACT

Uncertainties affect the accuracy of nonlinear static or dynamic optimization and inverse problems. The propagation of uncertain model parameters towards the optimal problem solutions can be assessed in a deterministic or stochastic way using Monte Carlo based techniques and efficient spectral collocation and Galerkin projection methods. This paper presents cost function transformations for reducing the impact of uncertain model parameters towards the optimal solutions. We assess the consistency of the methodology by determining sufficient conditions on the cost function transformations and apply the methodology on several test functions.

1. Introduction

Noise in measurement data and modeling errors leads to a deterioration of the optimal solutions' accuracy in optimal design problems and inverse problems. The impact of model parameters on the given static or dynamic model's outputs, can be assessed by performing e.g. a sensitivity or Monte Carlo analysis. Efforts have been dedicated at reducing the computational effort like the use of the surface response method applied in e.g. [1]; or the use of interval analysis [2] and polynomial chaos decomposition [3]. So-called intrusive methods such as the stochastic finite element method calculate the effect of stochastic parameters on the response by altering the calculation methodology itself. The spectral collocation method [4] or the spectral Galerkin projection method [5] allows to efficiently calculate the statistical moments such as the mean and variance; and the sensitivity information in advance [6].

Electromagnetic devices typically exhibit uncertainties towards uncertain material, geometrical or source parameters since they may contain parameter values that are difficult to determine or whose value is known to be situated within a certain range. The demand for robustness is often neglected during the optimization process and often worst parameter combinations of a design setting that is subject to uncertainty is identified [7]. Such uncertain parameters also arise in inverse problems [8] and a possible strategy is to estimate the values of these parameters but is often intractable.

Next to assessing the impact of uncertainties on optimal solutions, optimization under uncertainty can be performed where robust optimization [9,10] and reliability-based optimization [11,12] are methods currently used and topics of research. Recently, techniques have been proposed that reduce the impact of uncertain model parameters in applications based

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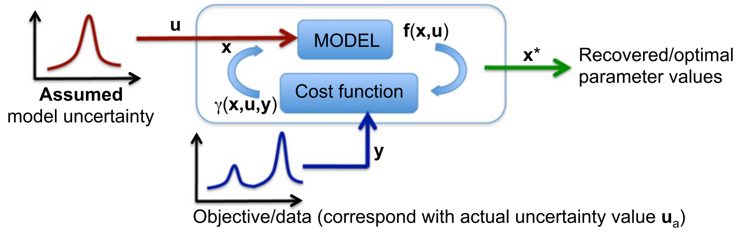

Fig. 1. Uncertainties affect the accuracy of optimal parameter values.

on electromagnetic principles: reduction of the uncertain electrical conductivity parameter in electroencephalographic source reconstruction [13]; reduction of the uncertain geometrical parameters on the magnetic material identification of an electromagnetic inductor [14]. Cost function formulations different than those considered in robust optimization and reliability-based optimization are presented in this paper where the defect correction principle is employed for performing optimization under uncertainty.

The work addresses the following specific class of problems: a vector $\mathbf{x} \in \Omega_x \subseteq \mathbb{R}^p$ needs to be estimated based on degraded data or objective $\mathbf{y} \in \Omega_y \subset \mathbb{R}^q$. Let $\mathbf{f} : \Omega_x \times \Omega_u \to \Omega_y$ denote the forward model that includes the physics of the system under study. Next to parameter values $\mathbf{x}$ to be estimated, $\mathbf{f}$ depends on uncertain parameters $\mathbf{u} \in \Omega_u \subseteq \mathbb{R}^r$. This paper discusses the propagation of $\mathbf{u}$ towards solutions in deterministic nonlinear least-squares problems. Fig. 1 illustrates the effect of assumed uncertain model uncertainties that are incorporated in a model and where starting from an objective or data, optimal parameter values are determined. Values for the parameter $\mathbf{u}$ that are assumed, and differ from the actual uncertainty parameter value, affect the values of the optimal parameters and thus the accuracy of the optimization process.

2. Cost function transformations

2.1. Propagation of uncertainties to the reconstructed parameters

The discrepancy between the aim and a particular response of the mathematical model is measured through the cost function:

$$\gamma(\mathbf{x}, \mathbf{u}, \mathbf{y}) = |||\mathbf{f}(\mathbf{x}, \mathbf{u}) - \mathbf{y}||| \tag{1}$$

for a certain defined $|||\cdot|||$ norm. An ℓ_2-norm formulation for the norm $|||\cdot|||$ is often used to find a best parameter value $\mathbf{x}^* \in \Omega_x$, which is a minimum of (1) over $\mathbf{x} \in \Omega_x$ for given objective $\mathbf{y}$ and assumed value $\mathbf{u}$ of the uncertainty parameter. Remark that we have no regularization term in the cost function (1). We address smooth models where $\mathbf{f}(\cdot, \cdot)$ is continuously differentiable towards $\mathbf{x}$ and $\mathbf{u}$. Cost function (1) needs reformulation in case a stochastic probability density function of uncertainties is provided. The discrepancy between statistical moments can then be used as cost function formulation.

Definition. The propagation function of the assumed uncertain model parameters towards the reconstructed parameters is defined as

$$\mathbf{x}^* : \Omega_u \times \Omega_y \to \Omega_x : (\mathbf{u}, \mathbf{y}) \mapsto \underset{\mathbf{x}\in\Omega_x}{\arg\min}\, \gamma(\mathbf{x}, \mathbf{u}, \mathbf{y}). \tag{2}$$

We furthermore assume in this paper that problem statement (2) is uniquely solvable $\forall \mathbf{u} \in \Omega_u, \forall \mathbf{y} \in \Omega_y$. This assumption holds in case of convex cost functions γ towards $\mathbf{x} \in \Omega_x$. The optimal solution parameter values $\mathbf{x}^*(\mathbf{u}_a, \mathbf{y})$ for actual value of the uncertainty parameter $\mathbf{u}_a$, may differ from the actual value of the sought after parameter value due to noise present in the data $\mathbf{y}$ or a not complete representation of reality in the model.

Definition. The error propagation function of the assumed uncertain model parameters towards the reconstructed parameters is defined as

$$e : \Omega_u \times \Omega_y \to \mathbb{R} : (\mathbf{u}, \mathbf{y}) \mapsto \left\| \mathbf{x}^*(\mathbf{u}, \mathbf{y}) - \mathbf{x}^*(\mathbf{u}_a, \mathbf{y}) \right\|_2. \tag{3}$$

The distance between the reconstructed parameter values is measured here using the ℓ_2-norm.

2.2. Transformations

The model corresponding with the actual uncertainty parameter value $\mathbf{u}_a$ is denoted by

$$\mathbf{h}(\mathbf{x}) \equiv \mathbf{f}(\mathbf{x}, \mathbf{u}_a), \tag{4}$$

whereas the model with assumed uncertainty parameter value $\tilde{\mathbf{u}}$ is given by

$$\mathbf{g}(\mathbf{x}) \equiv \mathbf{f}(\mathbf{x}, \tilde{\mathbf{u}}). \tag{5}$$