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Computation and application of Copula-based weighted average quantile regression

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HIGHLIGHTS

- We propose weighted average quantile method to estimate Copulas.
- Asymptotic properties of the estimators are provided.
- Simulations show satisfactory performance of the proposed estimates.
- The weighted average quantile Copula technique is applied to investigate the dependent structure and risk measurement of Chinese financial markets.

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ABSTRACT

Copula theories have obtained rapid development with respect to the dependent analysis of financial markets. The widely adopted technique for estimating Copula is maximum likelihood. In distinction, this paper proposes weighted average quantile regression to estimate Copula, which shares robustness from quantile regression and achieves nearly the same efficiency as the maximum likelihood. Consistency and asymptotic normality of the suggested estimators are established under regularity conditions. Through solving a quadratic programming, we obtain the expression of optimal weights. Monte Carlo simulations are conducted to compare the performance of different estimators. The proposed approach is used to investigate the dependent structure and risk measurement of Chinese financial markets.

1. Introduction

The research of dependent structure of risk elements is an essential process in risk management. The traditional analysis of dependency mainly adopts the linear measure. With the increasing complexity of financial systems, this index has failed to fully meet the requirements of risk management. There is an urgent demand for technology to deal with complex risk and nonlinear risk. In this context, Copula methods come into being as the times requires. Not only can it avoid the ineffectiveness of linear correlation coefficient in measuring dependence for financial data, but also it can model the joint distribution function of random variables by estimating marginals and Copula. These features of Copula provide new tools to study dependency.

Copula was proposed by Sklar [1] and had obtained a rapid development in the past ten years. Nelsen [2] elaborated and summarized the method of Copula. In terms of risk researches, Embrechts et al. [3] introduced Copula function to analyze financial risk and pointed out the limitations of linear measure. Patton [4] discussed the dependent structure between

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stock market and foreign exchange market with Copulas. Bedford [5] considered the Copula-based approach for studying a competing risk problem. He and Gong [6] constructed a Copula-based conditional Value-at-Risk model for market and credit risks. Using Archimedean Copulas, [7] examined the dependent structure between credit default swap and jump risk.

Maximum likelihood is the commonly used technique for estimating Copula. However, this method is cumbersome in computing and requires specifying the distributions of models. As one of the alternative technologies of model analysis, quantile regression proposed by Koenker and Bassett [8] has obtained dramatic attention and wide application. The main advantage of this method is that the stochastic relationship between random variables can be better portrayed on different quantiles, which provides more information than maximum likelihood method. For the details of quantile method, we can refer to [9]. The works of using quantile regression to analyze Copula are relatively scarce, compared with maximum likelihood. Bouyé and Salmon [10] first employed quantile regression to estimate Copula, but they did not give the asymptotic properties of estimators. Moreover, the asymptotic properties of Copulas based on quantile regression were established by Chen [11] whose proof was limited to the Markov framework.

This paper generalizes the quantile technique and proposes weighted average quantile method to estimate Copula. The estimator of weighted average quantile is a weighted sum of different quantile estimators. It shears robustness from quantile estimator and includes quantile estimator as a special case. Under some regularity conditions, we examine the asymptotic properties of proposed estimators which are proved to be consistent and asymptotically normal. The weighted average quantile regression with a data-driven weighting scheme is adaptive, which means that it performs as well as if the optimal weights were known. To fulfill this aim, we construct a quadratic programming to select the optimal weights. Simulations are conducted to evaluate the finite sample properties of different estimators. Application of the proposed methodology to discuss the dependency between Chinese Composite Indexes indicates the nonlinear relationship of China's financial markets. We also calculate the Value-at-Risk of portfolio based on the methods of Copula function and normal distribution respectively. It shows that the Copula method is dominate over the normal method.

The remainder of this paper begins with an illustration of Copulas and quantile functions in Section 2. Section 3 introduces the weighted average quantile Copula method. Monte Carlo simulations are given in Section 4. Section 5 is the application. All the proofs of theorems are arranged in the Appendix.

2. Copula and its quantile functions

Copula is a function which joins the marginal distribution functions to form a joint distribution function. Nelsen [2] gave the definition of bivariate Copula functions.

A function $C(\cdot)$: $[0,1]^2 \to [0,1]$ is a bivariate Copula, when the following conditions are satisfied.

(1) For any $u, v \in [0,1]$, $C(u,0) = C(0,v) = 0$, $C(u,1) = u$ and $C(1,v) = v$.
(2) For any $0 \le u_1 \le u_2 \le 1$ and $0 \le v_1 \le v_2 \le 1$,

$$C(u_2,v_2) - C(u_2,v_1) - C(u_1,v_2) + C(u_1,v_1) \ge 0.$$

Consider a random vector (X, Y) with continuous marginal distribution functions $F_X(x)$ and $F_Y(y)$, respectively. By Sklar's theorem [1], there exists a unique Copula $C(\cdot)$, such that $p(X \le x, Y \le y) = C\,(F_X(x), F_Y(y))$. Obviously, a bivariate Copula $C(\cdot)$ is a bivariate cumulative distribution function whose marginal distributions are uniformly distributed on $[0,1]^2$. From the definition of bivariate Copulas, the dimension of Copula functions can be generalized to any multidimensional cases.

Suppose that $F_{Y|X}(y|x)$ is the cumulative distribution function of Y conditional on $X = x$. Then, it is clear that

$$\begin{aligned} F_{Y|X}(y|x) &= p(Y \le y|X = x) \\ &= E\big(I(Y \le y)|X = x\big) \\ &= \lim_{\varsigma\to 0} p\big(Y \le y|x \le X \le x+\varsigma\big) \\ &= \lim_{\varsigma\to 0} \frac{F_{XY}(x+\varsigma, y) - F_{XY}(x,y)}{F_X(x+\varsigma) - F_X(x)} \\ &= \lim_{\varsigma\to 0} \frac{C\big(F_X(x+\varsigma), F_Y(y); \boldsymbol{\theta}\big) - C\big(F_X(x), F_Y(y); \boldsymbol{\theta}\big)}{F_X(x+\varsigma) - F_X(x)} \\ &= C_1\big(F_X(x), F_Y(y); \boldsymbol{\theta}\big), \end{aligned} \tag{1}$$

where $C_1(u,v;\boldsymbol{\theta}) = \partial C(u,v;\boldsymbol{\theta})/\partial u$ and $C(u,v;\boldsymbol{\theta})$ is a Copula function with parameters $\boldsymbol{\theta}$. From the distributional theories, we know that both $F_X(x)$ and $F_Y(y)$ are nondecreasing, which means that $F_{Y|X}(y|x)$ is also nondecreasing on y.

For any $\tau \in (0,1)$, solving

$$\tau = F_{Y|X}(y|x) \equiv C_1\big(F_X(x), F_Y(y); \boldsymbol{\theta}\big), \tag{2}$$

we obtain the τth quantile function of Y given $X = x$:

$$Q_Y(\tau|X = x) = F_Y^{-1}\big(\Gamma(F_X(x), \tau; \boldsymbol{\theta})\big), \tag{3}$$

where Γ is the partial inverse in the second argument of C_1 and F_Y^{-1} signifies the inverse of F_Y.