Journal of Computational and Applied Mathematics 281 (2015) 207–229

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

An improved implicit re-initialization method for the level set function applied to shape and topology optimization of fluid

Xiaomin Liu *, Bin Zhang, Jinju Sun

School of Energy and Power Engineering, Xi'an Jiaotong University, Xi'an, Shaanxi, 710049, China

ARTICLE INFO

Article history:
Received 30 May 2012
Received in revised form 28 November 2014

Keywords:
Level set method
Re-initialization
Smoothing parameter
Shape and topology optimization

ABSTRACT

This paper presents an accurate implicit re-initialization approach in the framework of a level set method. The improved method includes two schemes for keeping the zero level set unperturbed. The first scheme is to derive and use a new formula for the smoothing parameter in the conventional re-initialization equation based on the principle of the interface not moving. The second scheme is to reduce the local time step to avoid the interface moving across grid points when the sign of the level set function near the interface is changed. The example presented suggests that the new algorithm has a better approximation to the signed distance function and obtains a more accurate interface than the algorithm presented by Sussman et al. (1994). For shape and topology optimization of fluid, the effectiveness of the developed method in this paper was demonstrated by the internal and external flow examples.

1. Introduction

Firstly devised by Osher and Sethian [1] in 1988, the level set method is a powerful tool to capture moving interface, which has been widely used in image processing, two-phase flow, crystallization, computational geometry and so on [2,3]. One key issue for the level set method [1,4] is how to reset the level set function to be a signed distance function without obviously moving the interface. This issue is known as the re-initialization of the level set function.

Chopp [5] firstly reported the idea of reinitializing the level set function to be a signed distance function in 1993. He found that the re-initialization process overcomes the numerical oscillation problem caused by the too large or too small gradient of the level set function in the process of the level set function evolution. Sussman et al. [6] proposed a partial differential equation that implicitly reinitialized the level set function to be a signed distance function. The differential equation is an important framework for other implicit re-initialization equations, so we called it the "conventional equation". Peng et al. [4] developed a new implicit re-initialization method by modifying the smoothing parameter value in the conventional equation which obtains a better re-initialization result when the level set function is flat or steep near the interface. Sussman et al. [7] also developed an improved method by adding a local constraint to the conventional equation. However, the improved method may produce unphysical oscillations in a case that two different compressible fluids are separated by a sharp interface [8]. Russo and Smereka [9] proposed a sub-cell-fix method by applying a real upwind discretization near the interface to the conventional equation in order to keep the original interface undisturbed. Min and Gibou [10] explored an accurate re-initialization process by implementing the sub-cell-fix method on non-graded adaptive Cartesian grids. The improvement schemes of Hartmann et al. [11,12] and Sun et al. [13] continued to increase the accuracy of the sub-cell-fix method for several challenging test cases. Another re-initialization method is the fast marching method. Adalsteinsson and

* Corresponding author. Tel.: +86 29 82663777 308.
E-mail address: liuxm@mail.xjtu.edu.cn (X. Liu).

http://dx.doi.org/10.1016/j.cam.2014.12.017
0377-0427/

Sethian [14] reported that the fast marching method can produce an effective re-initialization in the narrow band level set method. However, the exact location of the interface must be explicitly known in the fast marching method, which is different from the above implicit methods. On the other hand, although the fast marching method can guarantee the interface not appreciably move in the process of re-initialization when the finer mesh is adopted, the computational cost would greatly increase due to the huge amount of grids.

Except for the methods mentioned above, Chan [15] and Li [16] proposed a variational level set method which eliminates the re-initialization process. They corrected the level set function to a signed distance function by adding an internal energy term to the original level set equation. Duan et al. [17–19] developed the variational level set method and applied it to the shape-topology optimization for the Stokes and Navier–Stokes flow. Molchanov et al. [20] proposed a new non-iterative method for the approximation of signed distance functions which could be used to the signed distance functions for any iso-surface representation.

However, to the best knowledge of authors, there are rare reports in literature on reinitializing the level set function without moving the interface obviously by improving the smoothing parameter in the conventional equation. In this paper, to keep the interface unperturbed, we derive a new formula for the smoothing parameter of the level set function and reduce the local time step when the sign of the level set function is changed. Several numerical examples are used to illustrate the performance of the presented method and verify its effectiveness and formal accuracy. For demonstration purposes, in Section 6 of this paper, the improved method is also applied to topology optimization of Navier–Stokes flow.

2. The implicit re-initialization method

2.1. The level set function

Let $\Omega \subset \mathbb{R}^N$ be a bounded open set with a smooth boundary $\Gamma = \partial\Omega$ and D be a work domain of $\Omega \subset D$, we define the level set function $\phi(x, t)$ in D as follows:

$$\begin{cases} \phi(x,t) > 0 & \forall x \in \Omega \\ \phi(x,t) < 0 & \forall x \notin \Omega \\ \phi(x,t) = 0 & \forall x \in \Gamma(t) = \partial\Omega. \end{cases} \tag{1}$$

The basic idea for the level set function to capture the moving interface is that the low dimensional moving interface can be described by a high dimensional level set function. Many advantages of the level set method have been reported [3]: the geometric features such as the boundary and the curve are convenient to describe, topology changes can be achieved easily and the numerical implementation is stable.

In the process of level set function evolution, the free interface is expressed as

$$\phi(x,t) = 0. \tag{2}$$

By the chain rule for Eq. (2), we obtain

$$\frac{\partial\phi}{\partial t} + \nabla\phi \cdot x'(t) = 0. \tag{3}$$

Let $V_n = x'(t) \cdot n$ be the level set normal velocity and $n = \frac{\nabla\phi}{|\nabla\phi|}$ be the unit normal vector in the free interface, so we obtain

$$\frac{\partial\phi}{\partial t} + V_n|\nabla\phi| = 0. \tag{4}$$

The value of the level set function $\phi(x, t)$ is obtained by solving Eq. (4).

In order to avoid the numerical oscillation while solving Eq. (4) by a numerical method, we always hope that

$$0 < C_1 \le |\nabla\phi| \le C_2. \tag{5}$$

The appropriate values of C_1 and C_2 would effectively control the above numerical oscillation. It is ideal to let the level set function $\phi(x, t)$ be a signed distance function, that is

$$|\nabla\phi| = 1. \tag{6}$$

Even if the initial level set function is a signed distance function, it is difficult to keep its nature of level set function during evolution. Therefore, it is necessary to carry out the re-initialization process in the appropriate time.

2.2. The conventional implicit re-initialization method

For the spatial two-dimensional problem, the implicit method is used to solve the following partial differential equations

$$\frac{\partial\phi}{\partial t} = S(\phi)\left(1 - \sqrt{\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2}\right), \tag{7}$$

$$\phi(x,0) = \phi_0(x), \tag{8}$$