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Reliability of coherent systems with a single cold standby component

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ABSTRACT

In this paper, the influence of a cold standby component on a coherent system is studied. A method for computing the system reliability of coherent systems with a cold standby component based on signature is presented. Numerical examples are presented. Reliability and mean time to failure of different systems are computed.

1. Introduction

There are different methods to increase system reliability. One of them is to equip the system with standby units such as warm, hot and cold. Compared to others, cold standby redundancy can be preferred when switching times are sufficiently short, since cold standby component is inactive which means it does not fail in standby. Van Gemund and Reijns [1] studied k-out-of-n system with a single standby and found an analytical way to compute the mean time to failure of the system. Eryilmaz [2] investigated various mean residual life functions for the same system. Recently, Eryilmaz [3] studied k-out-of-n system equipped with a single warm standby component.

In this paper, using system signature, conditioning on the index of the cold standby component and indices of the components failed before cold standby component is put into operation, the reliability of coherent systems having a cold standby component is derived.

Let X_i denote the lifetime of the ith component in a coherent system having lifetime T. If X_i's are s-independent and have common absolutely continuous distribution function, then the survival function can be represented as

$$P(T > t) = \sum_{i=1}^{n} p_i P(X_{i:n} > t),$$

where $X_{1:n} \le X_{2:n} \le \cdots \le X_{n:n}$ are the order statistics associated with $X_1, X_2, \ldots, X_n$ and $p_i = P(T = X_{i:n})$, in other words,

$$p_i = \frac{\text{The number of orderings for which the } i\text{th failure causes the system failure}}{n!},$$

for $i = 1, 2, \ldots, n$ which is well known as Samaniego's Signature [4]. The ith element of the signature vector can be easily computed from

$$p_i = \frac{r_{n-i+1}(n)}{\binom{n}{n-i+1}} - \frac{r_{n-i}(n)}{\binom{n}{n-i}}, \quad \text{for } i = 1, 2, \ldots, n.$$

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[5] where $r_i(n)$ denotes thenumber of path sets including i working components in a coherent system. Since signature has vital importance to investigate the behavior of the system lifetime, recently many developments have been made in this area, see [6–13].

In a coherent system with a single cold standby, the index of the standby component as well as the indices of failed components has significant importance which makes the computation of the reliability more difficult. In Section 3, a method for computing the reliability of coherent systems is presented. Finally, comparison of the reliability and mean time to failure of some systems with and without cold standby component have been illustrated.

2. Notations

Below the notations that will be used throughout the article are provided.

n, number of components in the system;
Y, lifetime of the cold standby component;
X_i, lifetime of the component i, $1 \le i \le n$;
$X_{s:n}$, sth smallest among X_i, $1 \le i \le n$;
$X_l^{(s)}$, remaining lifetime of the components after $X_{s:n}$ fails: $X_l^{(s)} \overset{st}{=} (X_l - X_{s:n} | X_l > X_{s:n})$, $1 \le l \le n - s$;
ϕ, structure function of the system;
$T = \phi(X_1, \ldots, X_n)$, lifetime of the system without cold standby component;
T^w, lifetime of the system with a cold standby component;
V_s, discrete random variable representing the index of the cold standby component when $X_{s:n}$ fails: $V_s = c \Leftrightarrow (X_c = X_{s:n} | T = X_{s:n})$, $c = 1, 2, \ldots, n$;
$\mathbf{B}_{s,c}|V_s = c$, a discrete multivariate random variable representing the indices of the failed components given $V_s = c$, $s = 1, \ldots, n$ and $c = 1, \ldots, n$;
$(\mathbf{B}_{s,c}|V_s = c) = (B_1 = b_1, B_2 = b_2, \ldots, B_{s-1} = b_{s-1}|V_s = c) \Leftrightarrow (0_{B_1} = 0_{b_1}, 0_{B_2} = 0_{b_2}, \ldots, 0_{B_{s-1}} = 0_{b_{s-1}}|X_c = X_{s:n}, T = X_{s:n})$ where $\mathbf{0} = (0_{B_1}, 0_{B_2}, \ldots, 0_{B_{s-1}})$ are the components which have failed before $X_{s:n}$;
$\mathbf{R}_{s,c}|V_s = c$, a discrete multivariate random variable representing the indices of the remaining components given $V_s = c$, $s = 1, \ldots, n$ and $c = 1, \ldots, n$: $\mathbf{R}_{s,c} = (R_1 = r_1, R_2 = r_2, \ldots, R_{n-s} = r_{n-s}|V_s = c) \Leftrightarrow (X_{R_1}^{(s)} = X_{r_1}^{(s)}, X_{R_2}^{(s)} = X_{r_2}^{(s)}, \ldots, X_{R_{n-s}}^{(s)} = X_{r_{n-s}}^{(s)} | X_c = X_{s:n}, T = X_{s:n})$.

3. Main results

Consider a binary coherent system with structure function ϕ. Let $T = \phi(X_1, \ldots, X_n)$ denote the lifetime of a coherent system without a cold standby component and T^w denote the lifetime of the same system with a cold standby component whose lifetime is Y. Moreover, $X_1, \ldots, X_n$ have a common continuous cumulative distribution function (c.d.f); F and Y have a continuous c.d.f G.

Eryilmaz [14] studied coherent systems equipped with a cold standby component which may be put into operation at the time of the first component failure in the system. In this paper, we consider the general case in which the standby component may get involved at the time of the sth component failure $s = k_\phi, \ldots, z_\phi + 1$ where k_ϕ is the minimum number of failed components that cause the system failure whereas z_ϕ is the maximum number of failed components that system can still operate. It is clear that $P(T = X_{s:n}) > 0$ for $s = k_\phi, \ldots, z_\phi + 1$.

After replacing the standby component with sth failed component which causes the system failure at the same time, the remaining lifetime of the system consisting of $s - 1$ failed components (0's), $n - s$ functioning components, and a standby component (Y) can be represented as

$$\phi_s(0_{B_1}, 0_{B_2}, \ldots, 0_{B_{s-1}}, Y_{V_s}, X_{R_1}^{(s)}, X_{R_2}^{(s)}, \ldots, X_{R_{n-s}}^{(s)}).$$

When sth failure occurs which causes system failure at the same time, cold standby component gets involved to the system. At this time, there are totally $n - s + 1$ functioning components in the system. The reliability of the remaining lifetime of the system is computed based on these $n - s + 1$ functioning components. However, places of the $s - 1$ failed components should be taken into consideration (not their lifetimes since they failed already) in the structure function of the system to calculate the main lifetime random variable T^w.

It is well known that the random variables $X_1^{(s)}, \ldots, X_{n-s}^{(s)}$ are conditionally independent given $X_{s:n} = x$, and

$$P\{X_1^{(s)} > x_1, \ldots, X_{n-s}^{(s)} > x_{n-s} | X_{s:n} = x\} = \prod_{l=1}^{n-s} \frac{\bar{F}(x_l + x)}{\bar{F}(x)}.$$

The main goal is to find the reliability characteristics of T^w, i.e.

$$T^w = T + \sum_{s=k_\phi}^{z_\phi+1} \phi_s(0_{B_1}, 0_{B_2}, \ldots, 0_{B_{s-1}}, Y_{V_s}, X_{R_1}^{(s)}, X_{R_2}^{(s)}, \ldots, X_{R_{n-s}}^{(s)}).$$