Journal of Computational and Applied Mathematics 271 (2014) 20–30

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

A quasi-interpolation scheme for periodic data based on multiquadric trigonometric B-splines[☆]

Wenwu Gao[a,b], Zongmin Wu[b,*]

[a] *School of Economics, Anhui University, Hefei, PR China*

[b] *Shanghai Key Laboratory for Contemporary Applied Mathematics, School of Mathematical Sciences, Fudan University, Shanghai, PR China*

ARTICLE INFO

Article history:
Received 28 May 2013
Received in revised form 3 March 2014

MSC:
41A25
41A30
65D10
65D15

Keywords:
Quasi-interpolation
Periodic data
Trigonometric divided differences
Multiquadric trigonometric B-splines
Active contours

ABSTRACT

Multiquadric (MQ) quasi-interpolation has been studied extensively in the literature. However, the MQ quasi-interpolant is not well defined for periodic data, since its kernel (the MQ function) itself is not periodic. Note that in many applications, the data may arise from a closed curve (surface) and thus possess some kind of periodicity, for example, analysis of geodetic and meteorological data, constructing active contours, estimating the region of attraction of dynamical systems, and so on. Therefore, it is meaningful to construct a quasi-interpolant (whose kernel itself is periodic) for periodic data. Combining the construction techniques of the MQ quasi-interpolant and the trigonometric B-spline quasi-interpolant, the paper constructs such a quasi-interpolant. The quasi-interpolant couples together the periodicity of the trigonometric B-spline quasi-interpolant and the smoothness of the MQ quasi-interpolant. Moreover, both the quasi-interpolant and its derivatives are periodic. The quasi-interpolant covers the trigonometric B-spline quasi-interpolant as a special case. In addition, the error estimate shows that a proper shape parameter can be chosen such that the quasi-interpolant provides the same approximation order as a trigonometric B-spline quasi-interpolant for a periodic function. Furthermore, the quasi-interpolant gives better approximations to high-order derivatives than the trigonometric B-spline quasi-interpolant, as illustrated by numerical examples presented at the end of the paper.

Contents

☆ This work is supported by SGST 12DZ 2272800.

* Corresponding author.
E-mail addresses: wenwugao528@163.com (W. Gao), zmwu@fudan.edu.cn (Z. Wu).

http://dx.doi.org/10.1016/j.cam.2014.03.012

1. Introduction

Periodic data exist widely in practical applications. Examples include reconstructing any simply connected closed curve/surface (i.e., constructing active contours, estimating the region of attraction of dynamical systems, geometric modeling, etc.), fitting data collected at the Earth's surface (i.e., the geodetic and meteorological data, the seismic data, etc.), studying the circadian rhythm of hormones, hydrogen atoms, the solar corona, and so on.

Interpolation is a common tool to deal with the periodic data. Precioso et al. [1] used B-splines to construct active contours for image segmentations. Giesl [2] provided a thorough discussion on estimating the region of attraction of dynamical systems with radial basis function interpolation. Schoenberg [3] used trigonometric B-splines to interpolate periodic data.

But interpolation requires solving a large-scale linear system of equations, which is time consuming. Moreover, the sampling data may be noised in some cases, and interpolation cannot deal with noised data well. For improving the speed of computation and filtering the noise in the data, people usually use quasi-interpolation.

Quasi-interpolation has been widely discussed in the literature, see [4–10] and the references therein. The major advantage of quasi-interpolation is that it yields a solution directly without the need to solve any linear system of equations.

Based on the MQ function proposed by Hardy [11], Beatson and Powell [12] first constructed three univariate MQ quasi-interpolants to scattered centers from a bounded interval. An improved MQ quasi-interpolant was provided in [13]. They also discussed some shape preserving properties of these MQ quasi-interpolants. By introducing the definition of MQ B-splines, Beatson and Dyn [14] discussed MQ quasi-interpolation theoretically. For applications of MQ quasi-interpolation in numerical solutions of differential equations, we refer readers to [15,16] and the references therein.

The MQ quasi-interpolant is smooth, efficient and easy to compute. Moreover, it approximates high-order derivatives well [17] and requires only computing the second-order divided differences of the MQ function, and thus is simple and stable [18].

However, the MQ quasi-interpolant is not suitable for periodic data directly, since its kernel (the MQ function) itself is not periodic.

Using the technique of periodic extensions, a MQ quasi-interpolation scheme for periodic data was constructed in [15]. But periodic extensions require boundary conditions, which may be extremely complicated in many cases. Moreover, periodic extensions yield unwanted high-order discontinuous points at the boundaries, and thus destroy high-order smoothness of the approximand (see Table 6 and the red curve in Fig. 2 in Section 4 for an example).

For periodic data, it is better to construct a quasi-interpolation scheme whose kernel itself is also periodic.

Lyche, Schumaker and Stanley [19] constructed a quasi-interpolation scheme based on trigonometric B-splines [20] and studied its approximation orders for high-order derivatives. For more applications of the quasi-interpolation scheme, see [21,22] for instance.

The trigonometric B-spline quasi-interpolant is well suited for periodic data, since its basis (trigonometric B-splines) themselves are periodic, as pointed out by Lyche [23]. But, analogous to B-splines [24], the smoothness order of trigonometric B-splines is only $n - 2$, where n is the order of trigonometric B-splines. This implies that we have to use high-order trigonometric B-splines for approximating high-order derivatives in some cases (e.g., constructing active contours, numerical solutions of differential equations, etc.).

Motivated by the above discussions, we construct in this paper a quasi-interpolant that couples together the periodicity of the trigonometric B-spline quasi-interpolant and the smoothness of the MQ quasi-interpolant.

The construction of the quasi-interpolant consists of three steps. We first construct a periodic kernel based on the MQ function. Then, applying trigonometric divided differences to the periodic kernel gives the multiquadric trigonometric B-splines. Finally, with these multiquadric trigonometric B-splines being the basis, the quasi-interpolant is constructed.

The quasi-interpolant preserves many fair properties of MQ quasi-interpolant such as smoothness, simplicity, efficiency, capabilities of approximating high-order derivatives and so on. Moreover, the quasi-interpolant as well as its derivatives are periodic. The quasi-interpolant covers the trigonometric B-spline quasi-interpolant as a special case. In addition, one can choose a proper shape parameter such that it provides the same approximation order as the trigonometric B-spline quasi-interpolant for a periodic function. Furthermore, the quasi-interpolant gives better approximations to high-order derivatives than the trigonometric B-spline quasi-interpolant.

The paper is organized as follows.

To make the paper self-contained, some preliminaries about trigonometric divided differences and the trigonometric B-spline quasi-interpolation scheme are presented in Section 2. In Section 3, we construct the main quasi-interpolation scheme (3.3) and we derive its error estimates (see the Theorem 3.1). Numerical examples of applying the scheme in approximating a periodic function, the first-order and the second-order derivatives of the function, respectively, are presented in Section 4. Finally, conclusions and discussions are given in Section 5.

2. Preliminaries

We start by introducing the definition of trigonometric divided differences.