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A framework for robust measurement of implied correlation

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HIGHLIGHTS

- We determine correlation estimates by matching the observed index option price with its model price.
- Different basket price approximations lead to different implied correlation estimates.
- The industry standard for measuring implied correlation is a bad estimate when it is needed the most.
- We propose a new approach for determining accurately implied correlations.

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ABSTRACT

In this paper we consider the problem of deriving correlation estimates from observed option data. An implied correlation estimate arises when we match the observed index option price with a corresponding model price. The underlying model assumes that stock prices can be described using a lognormal distribution, while a Gaussian copula describes the dependence structure. Within this multivariate stock price model, the index option price is not given in a closed form and has to be approximated. Different methods exist and each choice leads to another implied correlation estimate.

We show that the traditional approach for determining implied correlations is a member of our more general framework. It turns out that the traditional implied correlation underestimates the real correlation. This error is more pronounced when some stock volatilities are large compared to the other volatility levels. We propose a new approach to measure implied correlation which does not have this drawback. However, our numerical illustrations show that determining implied correlations with the traditional approach may be justified for strike prices which are close to the at-the-money strike price.

We also show that implied correlation estimates can be used to define an index, called the Implied Correlation Index (ICX), which reflects the market's perception about future (short-term) co-movement between stock prices. Using a volatility index together with the ICX gives an accurate description of the current level of market fear.

1. Introduction

The increased activity in multi-asset derivatives has resulted in an increased exposure to correlation risk for financial institutions. As a consequence, the correlation between different assets is an important input variable for determining portfolio risk measures like the VaR. In [1,2], the authors investigate the impact of a misspecification of the correlation on various risk measures. It is shown that even small errors in the correlation estimates can lead to serious errors in the value of the VaR.

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Asset correlations are varying over time and a sudden change in the correlation levels may have an important impact on the risk profile of the asset portfolio; see e.g. [3] or [4]. Therefore, it is important to track these correlation levels over time, because it gives information about the level of diversification one can obtain by composing an asset portfolio. In this paper we build a framework for determining today's level of diversification possible in a basket of stocks. It is well-documented that in a market in distress, correlations are relatively high, indicating that the diversification benefit one can obtain by composing a portfolio is evaporating. In the extreme case there is no diversification possible and 'stock picking' does not make sense anymore; the portfolio return is not determined by the particular stocks composing the portfolio, but whether one is exposed to the stock market or not. The stock prices are moving in unison and the market behaves as one big asset.

Having an idea of today's diversification level is a challenging task. An estimate based on historical time series will result in a backward looking measure and can never contain information about an event which is not captured in the data set. Stock and index options are publicly traded instruments and these derivative prices contain information about the market's view about the future movements of the financial market. Backing out correlation estimates from these traded derivatives results in a *forward-looking* estimate of the future level of diversification, to which we refer as the *implied correlation*. A similar approach was followed in [5,6].

The traditional approach for determining implied correlations between stocks composing an index is based on a lognormal approximation for the distribution of the index. This methodology is widely adopted because it results in an easy and analytical expression for the implied correlation which is a combination of implied stock and index volatilities, provided that we change the weights of the index in an appropriate way. Knowledge about the implied level of correlation is specially needed in volatile times. The lognormal approximation tends to fail in these situations, which makes it a *bad estimate in times when it is needed the most*. Indeed, we show that the traditional approach results in a systematic underestimation of the true correlation parameter. The error becomes more pronounced when some of the stock volatilities become large. As a result, one has to be careful with the use of the traditional implied correlation.

We construct a framework for deriving implied correlation estimates. The implied correlation is determined by matching the observed index option price with a corresponding model price. This model assumes that the stock price dynamics can be described by a multivariate Black & Scholes model. Although this model suffers from some major drawbacks, it pays to consider this stock price model, because it is the most straightforward multivariate extension of the one-dimensional Black & Scholes model; different Brownian motions are connected by a Gaussian copula resulting in a parsimonious multivariate stock price model where each stock is described by one volatility parameter and the dependence is fully captured by the pairwise correlations. The multivariate Black & Scholes index option pricing formula can be considered as a benchmark pricing formula, similar to the one-dimensional Black & Scholes formula. In reality, implied stock volatilities will not be constant, which reflects the fact that a single volatility parameter is not sufficient to fully capture the stock dynamics. We account for this departure from the Black & Scholes setting by using the whole volatility surface. We also show that our new implied correlation can be used to construct an *Implied Correlation Index (ICX)* which is a more accurate co-movement measure than the current market standard for determining implied correlations, which is described in [7].

We show that the traditional approach for determining implied correlation estimates is a particular member of our general framework. However, within this framework, more accurate correlation measures can be investigated. For example, we consider a correlation estimate based on convex upper and lower bounds. These convex bounds have proven their efficiency in various actuarial and financial problems; see e.g. [8,9], among others. This new correlation index is more accurate and reliable than the traditional approach. In a regime where some volatilities are large, the new implied correlation still gives an accurate picture of the mean level of co-movement between the different stocks. Using implied correlation estimates for the period January 2000–October 2009, we find that the traditional approach always underestimates the real correlation levels, but the error remains small and is on average 1.5%. We conclude that the traditional approach is justified when determining at-the-money implied correlation levels. However, when deep out-of-the-money options are used, the traditional approach sometimes underestimates the level of correlation by 6%.

The paper is organized as follows. In Section 2, we introduce the financial market and the multivariate Black & Scholes model. In Section 3 the implied correlation smile and the Implied Correlation Index are introduced. After describing the most important numerical issues in Section 4, an illustration of the obtained results is given in Section 5. We show how the Implied Correlation Index behaves during the period January 2000 and October 2009. Market participants often use volatility indices as indicators for market fear. We argue that combining an estimate for the degree of co-movement with an estimate for the level of volatility results in a more accurate description of the concept market fear. Section 6 concludes the paper.

2. The financial market

We assume a financial market[1] where n different (dividend or non-dividend paying) stocks, labeled from 1 to n, are traded. The financial market is arbitrage-free and there exists a pricing measure $\mathbb{Q}$, equivalent to the physical probability measure $\mathbb{P}$, such that the current price of any pay-off at time T can be represented as the expectation of the discounted

[1] We use the common approach to describe the financial market via a filtered probability space $\left(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0\leq t\leq T}, \mathbb{P}\right)$.