Journal of Computational and Applied Mathematics 271 (2014) 100–116

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

A bootstrapping market implied moment matching calibration for models with time-dependent parameters

Florence Guillaume [a,*], Wim Schoutens [b]

[a] University of Antwerp, Department of Mathematics and Computer Sciences, Middelheimlaan 1, B-2020 Antwerpen, Belgium
[b] K.U. Leuven, Department of Mathematics, Celestijnenlaan 200 B, B-3001 Leuven, Belgium

HIGHLIGHTS

- Calibration of Markov models with piecewise constant parameters.
- Use of the additive property of cumulants to perform a sequential calibration.
- Comparison of the new calibration methodology with standard calibration procedures of term structure models.
- Evidence of the supremacy of the bootstrapping calibration in terms of the matching of the moment term structure.

ARTICLE INFO

Article history:
Received 21 June 2013
Received in revised form 31 March 2014

Keywords:
Market implied calibration
Moment matching
Bootstrapping calibration
Piecewise constant parameters
Term structure Lévy models

ABSTRACT

This paper extends the moment matching market implied calibration procedure (Guillaume and Schoutens 2012) to Markov models with piecewise constant parameters between successive quoted option maturities. The Markov property allows us to determine the parameter set of each subprocess by a bootstrapping moment matching calibration. This sequential calibration arises naturally due to the additive property of cumulants of independent random variables and consists in solving M independent moment matching systems of N equations, where M and N denote the number of quoted maturities and the number of parameters, respectively. As shown in Guillaume and Schoutens (2012), for popular Lévy processes, these systems can be transformed into M systems of algebraic equations which give directly the N model parameters of each subprocess in terms of the second to the $(N+1)$th standardized moments of the log asset return process between successive maturity times. For the numerical study, we work out the bootstrapping moment matching calibration under two popular Lévy models with piecewise constant parameters, namely the VG and Meixner models and compare its performance with existing calibration procedures for term structure models.

1. Introduction

The spanning option payoff formula of Breeden and Litzenberger [1,2] for twice-differentiable payoffs allows us to derive a model independent formula for any moment of the log asset return distribution function for a time horizon T, which is expressed as a function of a continuum of vanilla option prices (see [3]). In particular, we can infer an approximation for any moment in terms of liquid vanilla option prices with maturity time T. As long as the characteristic function of the log asset return is known in closed-form for the model under consideration, these formulas lead to an almost immediate

* Corresponding author. Tel.: +32 497270303.
E-mail addresses: florence.guillaume@uantwerpen.be (F. Guillaume), Wim@Schoutens.be (W. Schoutens).

http://dx.doi.org/10.1016/j.cam.2014.04.005
0377-0427/

market implied calibration, which consists in matching the market implied and the model moments for a time horizon T, chosen among the set of quoted maturities. In particular, it has been shown in [3] that, for popular Lévy models, the moment matching calibration problem reduces to a system of N equations which can be solved analytically and which gives thus directly the N optimal parameters in terms of the market implied standardized moments of order 2 to order $(N+1)$. Hence, the market implied moment matching procedure turns out to be appealing when calibrating exponential Lévy models on one single maturity curve. Indeed, since it only involves simple algebraic expressions, the moment calibration can be performed almost instantaneously and does not require starting values for the model parameters, avoiding thus the related problem of getting stuck in local minima.

Although suited to fit option quotes for one single maturity, it is well established that exponential Lévy models can lead to some discrepancy with the market reality once the whole set of maturities is considered. This empirical fact has been highlighted, among others, by Carr et al. [4] and by Guillaume [5]. Carr et al. considered a set of 21 largely traded stocks for which they compared the goodness of fit of the option price surface under various exponential Lévy and Sato models. In particular, they showed that Lévy processes are suited to replicate option prices for one single maturity but for the whole set of quoted maturities, Sato models typically outperform Lévy models. Guillaume has shown that multivariate Sato models outperform multivariate Lévy models regarding the marginal goodness of fit, especially during investor's fear periods. In particular, it is shown that Lévy models barely outperform the Black–Scholes model during the heart of the recent credit crisis. This implies that the moment term structure under exponential Lévy models might deviate significantly from the one observed in the market (see also [6]). Indeed, under such models, the variance, the skewness and the excess of kurtosis scale as T, $1/\sqrt{T}$ and $1/T$, respectively, whereas less parsimonious trends might be observed from option prices. Hence, although suited to calibrate exponential Lévy models on options with one single maturity, the market implied moment calibration procedure might lead to a significant discrepancy between the market implied and the model moments and, hence, to significant errors for a calibration on the whole set of maturities. Moreover, if the Lévy model does not allow for time-dependent parameters, we then have to resort to some search algorithm to find the optimal parameter set since we then have to solve an over-determinate system. The moment matching calibration then becomes an ill-posed problem and we are then faced with the related problem of choosing adequate starting values for the model parameters to avoid the problem of getting stuck in one bad local minimum of the objective function we minimize.

To better capture the term structure of the volatility smile, several alternative option pricing models have been proposed in the financial literature. Carr et al. [4] and Guillaume [5] proposed the so-called Sato models, built upon self-decomposable distributions, which constitute a sub-class of infinitely divisible distributions. Motivated by the evidence of time-dependent volatility, Kawai [7] proposed the so-called piecewise Lévy models, built in a similar fashion as the Lévy term structure models of Eberlein and Kluge [8] for interest rate derivatives, i.e. by allowing the parameters of the Lévy model to vary from one option maturity to the other. These term structure Lévy models can be seen as a sub-class of stochastic hybrid models where each transition map is only activated once and where the transition times, taken equal to the quoted option maturities, are deterministic rather than governed by some stochastic process. A third approach has been proposed by Madan and Konikov [6]. Motivated by the empirical evidence of volatility clustering, they built a stochastic hybrid model where the transitions between homogeneous parameter specifications of the Lévy process are governed by a Markov chain, allowing for moments clustering.

In this paper, we propose a bootstrapping extension of the moment matching calibration to calibrate models with time-dependent parameters, focusing therefore on the second class of alternative models. More particularly, we assume that the log asset returns over the periods $[T_{i-1}, T_i)$ are modeled by independent Lévy processes, where $\{T_i, i = 1, \dots, M\}$ denotes the set of quoted option maturities. Given the additive property of cumulants of independent random variables, we can calibrate such models by making use of a bootstrapping moment calibration procedure: we first determine successively the market implied moments of interest over the time periods $[T_{i-1}, T_i)$ and then solve the corresponding ith moment matching calibration problem for $i = 1, \dots, M$. For the numerical study, we will consider term structure exponential Lévy models built upon two popular infinitely divisible distributions, namely the VG and Meixner distributions.

This paper is organized as follows. Section 2 recalls the closed-form approximation of the moments of the log asset return derived in [3]. Section 3 details the term structure exponential Lévy models. In Section 4, we extend the moment matching calibration [3] by considering a bootstrapping methodology in order to calibrate models with time-dependent parameters. In Section 5, we work out the new bootstrapping moment matching calibration under popular term structure exponential Lévy models and compare its performance with other calibration procedures, namely the implied probability density method of Kawai [7] and the standard (bootstrapping) calibration on market quotes. Section 6 concludes.

2. Market implied moments: a closed-form formula

As shown in [3], the spanning option payoff formula of Breeden and Litzenberger [2] allows us to derive a closed-form approximation for any moment of the log asset return $X_T = \log\left(\frac{S_T}{S_0}\right)$:

$$\mathbb{E}\left[\log\left(\frac{S_T}{S_0}\right)\right] = \log\left(\frac{K_0}{S_0}\right) - 1 + \frac{F_0}{K_0} - \exp(rT)\sum_{i=1}^{M} \Delta K_i \frac{1}{K_i^2} Q(K_i). \tag{2.1}$$