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Stable, high order accurate adaptive schemes for long time, highly intermittent geophysics problems

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ABSTRACT

Many geophysical phenomena are characterized by properties that evolve over a wide range of scales which introduce difficulties when attempting to model these features in one computational method. We have developed a high-order finite difference method for the elastic wave equation that is able to efficiently handle varying temporal and spatial scales in a single, stand-alone framework. We apply this method to earthquake cycle models characterized by extremely long interseismic periods interspersed with abrupt, short periods of dynamic rupture. Through the use of summation-by-parts operators and weak enforcement of boundary conditions we derive a provably stable discretization. Time stepping is achieved through the implicit θ-method which allows us to take large time steps during the intermittent period between earthquakes and adapts appropriately to fully resolve rupture.

1. Introduction

Earthquake rupture is one example of many geophysical phenomena that are characterized by properties that evolve over many orders of magnitude in both time and space. Modeling these phenomena with full temporal and spatial resolution is thus quite difficult and it is often the case that simplifying assumptions are made in numerical studies. Because the initial conditions prior to an earthquake are not well understood, many studies of earthquake rupture for example, impose artificial initial conditions in the form of a stress perturbation in order to immediately nucleate dynamic rupture [1–3]. These methods capture the fine details of the rupture process and wave propagation, but are limited to single-earthquake simulations without realistic initial data.

Obtaining self-consistent initial conditions would require modeling the interseismic loading period prior to rupture, but this is computationally infeasible with the explicit time-stepping techniques generally used. Since stability considerations with explicit methods limit the size of the time step to fractions of a second, these methods are not appropriate for modeling the tectonic loading period characterized by tens to hundreds of years. In order to model full earthquake cycles however, these multiple time scales have been handled with several different techniques. The methods of [4] and [5] involve an abrupt switching between solving the static problem (in which inertia is neglected) and the dynamic problem. The method in [6] disregard inertia entirely and assume that the rupture is quasi-dynamic and therefore do not simulate wave propagation. The authors of [7] present a method that is able to simulate long interseismic periods punctuated by dynamic events within one computational framework, but the method is based on the boundary integral method and make the simplifying assumption of rupture occurring in a homogeneous, linear elastic whole or half-space.

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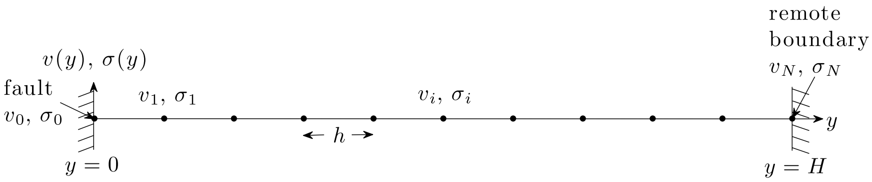

Fig. 1. Physical and computational setting for 1D elastic wave equation in first order form on an unstaggered grid. The system, initially essentially at rest, is loaded at the remote boundary $y = H$ by a velocity boundary condition intended to capture the effect of slow tectonic loading. Periodic earthquakes nucleate at the fault, which lies at the boundary $y = 0$ and is governed by a stress boundary condition. The domain is discretized at $N + 1$ points with grid spacing $h = H/N$.

In this work we simulate both the interseismic period and fully dynamic rupture in one-computational setting, with a volume discretization which allows the method to incorporate variable material properties. The method applies high order finite difference operators which provide an efficient approach, and yields a semi-discrete problem which is provably stable. The efficiency can be used either to increase the accuracy for a fixed number of mesh points or to reduce the computational cost for a given accuracy by reducing the number of mesh points [8,9]. In the past, the main drawback with high order finite difference methods was the complicated boundary treatment required to obtain a stable method. However, the development during the last two decades has removed this obstacle. Finite difference operators which satisfy the summation-by-parts (SBP) property [10–12], are central difference operators in the interior domain augmented with special stencils near the domain boundaries. These SBP operators in combination with weak well-posed boundary conditions lead to energy stability [13–19]. The preferred boundary treatment is the simultaneous approximation term (SAT) method [20], which linearly combines the partial differential equation to be solved with well-posed boundary conditions [21,13,22,23]. A complete description of the SBP–SAT method is given in the review article [24].

Time-stepping is done through the implicit θ-method which yields a first or second-order accurate (in time) method and is A-stable [25]. The time step adapts according to an estimate of the local truncation error, and can be quite large during the interseismic period while still maintaining stability. Although the main drawback compared to explicit methods is that a nonlinear system of equations must be solved at every time step, efficiency is gained by the ability to take large time steps, and we make no simplifying assumption of inertia being negligible during the interseismic period. Through this technique we obtain self-consistent initial conditions prior to rupture which reflect many years of tectonic loading. In this initial development we focus on the development of an efficient and stable time-stepping method for a high-order accurate spatial discretization. We consider the one-dimensional problem which contains all of the difficulties present in the multi-dimensional problem (such as varying temporal and spatial scales, and extreme stiffness), while providing the simplest possible framework in which to introduce the method. The extension to multi-dimensions is straight forward.

2. Continuous formulation and well-posedness

2.1. Preliminaries

We simulate multiple earthquake cycles where events nucleate at a frictional fault at one boundary of the domain. The material off the fault is governed by the elastic wave equation in first order form, see Fig. 1. In addition to the varying time scales governing geophysical phenomena, as described in the introduction, there are also computational challenges introduced through varying spatial scales. Faults can be hundreds of kilometers long, with frictional properties on the order of microns. These features often lead to very large problems in order to fully resolve multiple length scales.

2.2. Governing equations and well-posedness via the energy method

Assuming linear elasticity in first order form, the governing equations and boundary conditions are:

$$\frac{\partial w}{\partial t} = B\frac{\partial w}{\partial y}, \quad B = \begin{bmatrix} 0 & 1/\rho \\ \mu & 0 \end{bmatrix}, \; w = \begin{bmatrix} v \\ \sigma \end{bmatrix}, \; y \in [0, H] \tag{2.1a}$$

$$L_o(w) = \sigma(0, t) = F(V(t)), \qquad L_1(w) = v(H, t) = V_p. \tag{2.1b}$$

The parameters ρ and μ are the material density and shear modulus and the boundary operators L_o and L_1 act on the shear stress σ and particle velocity v, respectively. We assume that a frictional fault lies at $y = 0$ and is governed by a boundary condition that equates shear stress with fault strength given through an experimentally-motivated friction law F dependent on the particle velocity at the fault $V(t) = v(0, t)$ (known as the "slip velocity"), discussed in Section 4.3. The system is initially at rest and undergoes an interseismic period where it is loaded at the remote boundary. We set the velocity at the remote boundary $y = H$ to a slow "plate rate" V_p, intended to capture the effect of slow tectonic loading. Measurements of typical values of V_p are around 32 mm/a (e.g. the San Andreas Fault in southern California). This remote boundary condition will load the system and increase the stress at the fault, which will eventually cause earthquakes to initiate at the fault, sending waves through the medium.