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A new family of Marshall–Olkin extended distributions

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ABSTRACT

The purpose of this paper is to introduce, discuss and analyze a new family of Marshall–Olkin extended distributions. This new family generalizes the following Marshall–Olkin extended distributions: (i) exponential, (ii) Rayleigh, (iii) linear failure rate, and (iv) Weibull. Some statistical and reliability properties of the new family are discussed. The maximum likelihood estimates of its unknown parameters are obtained. The obtained results are validated using a real data set and it is shown that the new family provides a better fit than some other known distributions.

1. Introduction

Many of researchers are interested in search that introduces new families of distributions or generalized some of the presented distributions which can be used to describe the lifetimes of some devices or to describe sets of real data. Exponential, Rayleigh, Weibull and linear failure rate are some of the important distributions widely used in reliability theory and survival analysis. However, these distributions have a limited range of applicability and cannot represent all situations found in applications. For example, although the exponential distribution is often described as flexible, its hazard function is constant. The limitations of standard distributions often arouse the interest of researchers in finding new distributions by extending existing ones. The procedure of expanding a family of distributions for added flexibility or constructing covariates models is a well-known technique in the literature. For instance, the family of Weibull distributions contains exponential distribution and is constructed by taking powers of exponentially distributed random variables.

Marshall and Olkin [1] introduced a new method of adding a parameter into a family of distributions. The resulting distribution, known as Marshall–Olkin (M–O) extended distribution, includes the baseline distribution as a special case and gives more flexibility to model various types of data. According to them if $\overline{F}(x)$ denote the survival or reliability function of a continuous random variable X, then the timely honored device of adding a new parameter results in another survival function (SF) $\overline{G}(x)$ defined by

$$\overline{G}(x;\alpha) = \frac{\alpha \overline{F}(x)}{1-\overline{\alpha}\overline{F}(x)}, \quad -\infty \le x \le \infty,\ \alpha \ge 0, \overline{\alpha} = 1-\alpha. \tag{1}$$

The M–O extended distributions offer a wide range of behavior than the basic distributions from which they are derived. The property that the extended form of distributions can have an interesting hazard function depending on the value of the added parameter and therefore can be used to model real situations in a better manner than the basic distribution (cf. [1–13]). Let $h(x)$ and $r(x)$ denote the hazard rate functions of the extended distribution and the original distribution,

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respectively. Marshall and Olkin [1] have called the additional shape parameter "tilt parameter", since the hazard rate of the new family is shifted below ($\alpha > 1$) or above ($0 < \alpha \leq 1$) the hazard rate of the underlying distribution, that is, for all $x \geq 0$, $h(x) \leq r(x)$ when $\alpha > 1$, and $h(x) \geq r(x)$ when $0 < \alpha \leq 1$.

Marshall and Olkin [1] introduced a two-parameter extension of the exponential [M-OEE (α, σ)] distribution with SF

$$\overline{G}(x; \alpha, \sigma) = \frac{\alpha}{e^{\sigma x} - \overline{\alpha}}, \quad x > 0,\ \alpha, \sigma \geq 0,$$

three-parameter extension of the Weibull distribution [M-OEW (α, β, γ)] with SF

$$\overline{G}(x; \alpha, \beta, \gamma) = \frac{\alpha e^{-\beta x^{\gamma}}}{1 - \overline{\alpha} e^{-\beta x^{\gamma}}}, \quad x > 0,\ \alpha, \beta, \gamma > 0,$$

and two parameter extension of the Rayleigh distribution [M-OER (α, σ, β)] with SF

$$\overline{G}(x; \alpha, \beta) = \frac{\alpha e^{-\beta x^{2}}}{1 - \overline{\alpha} e^{-\beta x^{2}}}, \quad x > 0,\ \alpha, \sigma, \beta > 0.$$

In addition, based on the M–O method, Ghitany and Kotz [2] introduced and studied the extension of the linear failure rate distribution [M-OELFR (α, σ, β)] with SF

$$\overline{G}(x; \alpha, \sigma, \beta) = \frac{\alpha e^{-\sigma x - \beta x^{2}}}{1 - \overline{\alpha} e^{-\sigma x - \beta x^{2}}}, \quad x > 0,\ \alpha, \sigma, \beta > 0.$$

All the M–O distributions mentioned above are special cases of the new family that is introduced and studied in Section 2 below. In Section 3, some statistical and reliability properties of the new family are discussed. In Section 4, the method of maximum likelihood estimation is used to estimate the unknown parameters. In addition, simulation is utilized to calculate the unknown shape parameter and to study its properties. Section 5 gives some applications to explain how a real data set can be modeled by the new family. Finally, in Section 6, some conclusions and remarks of the current and future research are presented.

2. Density and hazard rate of the new family

Let X follows the modified Weibull (MW) distribution with SF

$$\overline{F}(x; \sigma, \beta, \gamma) = e^{-\sigma x - \beta x^{\gamma}}, \quad x > 0, \tag{2}$$

where $\gamma > 0, \sigma, \beta \geq 0$ such that $\sigma + \beta > 0$ (Sarhan and Zaindin [14]). Substituting (2) in (1) we get a new family of distributions called M–O extended modified Weibull distributions and denoted it as M-OEMW $(\alpha, \sigma, \beta, \gamma)$, with the following SF

$$\overline{G}(x; \alpha, \sigma, \beta, \gamma) = \frac{\alpha e^{-\sigma x - \beta x^{\gamma}}}{1 - \overline{\alpha} e^{-\sigma x - \beta x^{\gamma}}}, \quad x > 0,\ \alpha, \gamma > 0,\ \sigma, \beta \geq 0,\ \sigma + \beta > 0.$$

The corresponding cumulative distribution function (CDF) and probability distribution function (PDF) are obtained, respectively as follows:

$$G(x; \alpha, \sigma, \beta, \gamma) = \frac{1 - e^{-\sigma x - \beta x^{\gamma}}}{1 - \overline{\alpha} e^{-\sigma x - \beta x^{\gamma}}},$$

and

$$g(x; \alpha, \sigma, \beta, \gamma) = \frac{\alpha(\sigma + \beta\gamma x^{\gamma-1}) e^{-\sigma x - \beta x^{\gamma}}}{(1 - \overline{\alpha} e^{-\sigma x - \beta x^{\gamma}})^2},$$

where $x > 0,\ \alpha, \gamma > 0,\ \sigma, \beta \geq 0,\ \sigma + \beta > 0$. See Fig. 1.

The next result provides the behavior of the PDF of the M-OEMW distribution and can be verified using elementary calculus.

Proposition 2.1. *Let $X \sim$ M-OEMW $(\alpha, \sigma, \beta, \gamma)$; then X has*

(i) *increasing PDF provided $\overline{\alpha} < e^{\sigma x + \beta x^{\gamma}}$,*
(ii) *decreasing PDF provided $\overline{\alpha} > e^{\sigma x + \beta x^{\gamma}}$.*

The following example shows that the new family generalizes the M-O extended distributions of exponential, Weibull, Rayleigh and linear failure rate.

Example 2.1. Let $X \sim$ M-OEMW $(\alpha, \sigma, \beta, \gamma)$. Then

(i) if $\beta = 0$, then $X \sim$ M-OEE (α, σ);
(ii) if $\sigma = 0$, then $X \sim$ M-OEW (α, β, γ);
(iii) if $\sigma = 0$ and $\gamma = 2$, then $X \sim$ M-OER (α, σ, β);
(iv) if $\gamma = 2$, then $X \sim$ M-OELFR (α, σ, β).