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Default probabilities of a holding company, with complete and partial information

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ABSTRACT

This paper studies the valuation of credit risk for firms that own several subsidiaries or business lines. We provide simple analytical approximating expressions for probabilities of default, and for equity–debt market values, both in the case when the information is available in continuous time as well as in the case that it is not instantaneously available. The total firm's asset value being modeled as a sum of lognormal random variables, we use convex upper and lower approximations to infer these analytical approximating expressions. We extend the model to firms financed by multiple stochastic liabilities and conclude by numerical illustrations.

1. Introduction

The treatment of default is a crucial issue in determining the value of corporate securities and the firm's financing decisions. This task is particularly complex when the corporation is itself a group of subsidiaries that have dependent activities. Structural models such as developed by Merton [1] and Black and Cox [2] represent an elegant framework for the valuation of risky debts, when assets are modeled by a single Brownian motion. Since, many alternatives have been developed to replace the Brownian motion by more complex dynamics. Recently Fiorani [3], Ballotta and Fusai [4] and Hainaut and Colwell [5] used Lévy, multivariate Lévy and switching Lévy processes. But, up to our knowledge, there are only very few extensions to multi-industry firms.

Moreover, most of the existing models assume that the dynamics of firm's assets are continuously observed while in practice, the information needed to assess efficiently the financial health is for most of the companies only released at discrete times. As emphasized by Duffie and Lando [6], ignoring this aspect leads to an underestimation of short-term credit spreads.

The purpose of this work is hence twofold. Firstly, this paper proposes simple approximating formulas to appraise default probabilities, risky debts and equity for multi-industry firms. The total firm's asset is a sum of lognormal processes, each one corresponding to a firm's subsidiary. The statistical distribution of firm's total asset value exhibits then more leptokurticity and asymmetry than a single lognormal variable. Secondly, it studies the impact of a lack of information on these quantities. The framework of our model is partly inspired from papers of Leland [7,8] and of Leland and Toft [9]. We assume that the default or simply the restructuring events are triggered when the total market value of all subsidiaries falls below a certain threshold. Two cases are considered. In the first one, this threshold is constant. It can eventually be regulatory imposed, or chosen by the firm's management. In the second case, the threshold is random and the sum of several liabilities. This approach is particularly well adapted for insurance companies that finance their investments by e.g. life or non life provisions.

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The solution that we propose is based on the concept of convex orders and comonotonicity, which were introduced by Hoefdding [10] and Frechet [11] who studied lower and upper bounds for multivariate cumulative distributions. This theory became popular amongst researchers in actuarial sciences over the last two decades and has been applied successfully to various fields of research. Dhaene et al. [12,13] proposed in their review comonotone upper and lower approximations in the convex order sense for the sum of a finite number of random variables. The work of Vanduffel et al. [14] reveals that the lower convex bound approximation is extremely accurate for an appropriate choice of parameters. We refer the interested reader to [15] for characterizations of convex orders. We further notice that convex orders and stop loss premiums are closely related (see e.g. [16]). Comonotone bounds have been applied from derivatives pricing [17] to insurance [18], including risk management, as in [19]. For a recent survey of applications in finance and insurance, we refer e.g. to [20]. But we did not encounter any applications of this theory to the valuation of credit risk. Our work tries to fill this gap.

The outline of this paper is as follows. Section 2 introduces the framework that we adopt to model a multi activity firm. In Section 3, we build the convex bounds of the total firm's asset and infer in Section 4 approximating formulae for the probabilities of default. In Sections 5 and 6, we respectively appraise the value of debts with complete and incomplete information. In Section 7, the model is adapted to stochastic liabilities. Section 8 contains several numerical applications and we conclude our work in Section 9.

2. The model

We consider a holding company, composed of N subsidiaries or various business lines. Each subsidiary generates a stream of dividends or cash-flows distributed in its entirety to the parent company. The investment in the subsidiary is assumed irreversible, at least till an eventual restructuring of the holding. Dividends are defined on a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where $\mathcal{F}$ is the filtration generated by M independent Brownian motions, denoted by $\tilde{W}^j$ for $j = 1 \dots M$. The dividend provided by the ith subsidiary is assumed to be a stochastic process denoted by F_t^i and has the following dynamics under the real measure $\mathbb{P}$:

$$\frac{dF_t^i}{F_t^i} = \mu_i dt + \sum_{j=1}^{M} \sigma_{i,j} d\tilde{W}_t^j \quad \forall i = 1, \dots, N. \tag{2.1}$$

We denote Σ the $N \times M$ matrix of $\left(\sigma_{i,j}\right)_{i=1\dots N, j=1\dots M}$ which are such that $rank(\Sigma) = M$. The covariance matrix containing the covariances between the flows of dividends is then equal to $\Sigma\Sigma^\top$. We also assume that there exists a risk free asset, such as a bank account, that provides a constant rate of return r. The total flow of dividends paid to the holding company is denoted by

$$F_t = \sum_{i=1}^{N} F_t^i.$$

Obviously, cash-flow processes are not tradeable assets. However, as the subsidiary is a separate entity, the entire value of this subsidiary can be seen as a traded asset. In this case, the value of the ith subsidiary is equal to the expected sum of cash-flows discounted at the cost of the equity r_E with $r_E > \mu_i$:

$$\begin{aligned} S_t^i &= \mathbb{E}\left(\int_t^\infty e^{-r_E(s-t)} F_s^i ds \mid \mathcal{F}_t\right) \\ &= \int_t^\infty e^{-r_E(s-t)} \mathbb{E}\left(F_t^i e^{\left(\mu_i - \sum_{j=1}^{M} \frac{\sigma_{i,j}^2}{2}\right)(s-t) + \sum_{j=1}^{M} \sigma_{i,j}(W_s^j - W_t^j)} ds \mid \mathcal{F}_t\right) \\ &= \frac{F_t^i}{r_E - \mu_i}. \end{aligned} \tag{2.2}$$

This last formula is similar to the Gordon–Shapiro formula as first exposed by Gordon and Myron [21]. The rate μ_i can be seen as the growth rate of dividends distributed by the ith business line. Note that the expectation in Eq. (2.2) is calculated under the real measure $\mathbb{P}$, and this is the reason why the discount rate is given by the cost of equity and not by the risk free rate. The dynamics of S_t^i can be rewritten as

$$dS_t^i = \frac{dF_t^i}{r_E - \mu_i} = \mu_i S_t^i dt + S_t^i \sum_{j=1}^{M} \sigma_{i,j} d\tilde{W}_t^j. \tag{2.3}$$

Let us denote by μ, the vector of μ_i and by κ, the vector of market risk premiums κ_j for $\tilde{W}_t^i$. If $\mathbf{1}_N$ is a N vector of ones, κ is a solution (not necessary unique) of $\mu = \mathbf{1_N} r + \Sigma\kappa$. Then the risk neutral $\mathbb{Q}$ is defined by the following Radon–Nikodym derivative

$$\left(\frac{d\mathbb{Q}}{d\mathbb{P}}\right)_t = \exp\left(-\frac{1}{2}\int_0^t \kappa'\kappa ds - \int_0^t \kappa' d\tilde{W}_s\right). \tag{2.4}$$