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A superlinearly convergent SQP method without boundedness assumptions on any of the iterative sequences[☆]

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ABSTRACT

This paper is aimed to present a new sequential quadratic programming (SQP) algorithm for finding a solution to nonlinear constrained programming problems with weak conditions, where the improved direction can be yielded by solving one quadratic programming (QP), and the correction direction can be obtained by solving another QP. The main characters of the proposed algorithm are as follows. First, by limiting infeasibility of SQP iterates, the boundedness of the iteration sequence can be obtained in the case of the feasible set being nonempty and bounded as well as the constraint functions being convex. Second, global convergence can be proved under Slater constraint qualification (CQ). Furthermore, superlinear convergence can be ensured under suitable conditions. Third, the proposed algorithm is further improved with a bidirectional line search technique. Finally, some numerical experiments are operated to test the proposed algorithms, and the results demonstrate that they are promising.

1. Introduction

In this work, we consider the following nonlinear inequality constrained optimization:

$$\begin{aligned} &\min f(x) \\ &\text{s.t. } g_i(x) \le 0, \quad i \in I = \{1, 2, \dots, m\}, \end{aligned} \tag{1.1}$$

where the functions f and $g_i (i \in I) : \mathbb{R}^n \to \mathbb{R}$ are all at least continuously differentiable. It is well known that the SQP method is a kind of efficient approaches to solve problem (1.1), since it enjoys good superlinear convergence property, see Refs. [1–4]. In the traditional SQP methods, for the current iteration point $x^k \in \mathbb{R}^n$, to obtain the next iteration point x^{k+1}, the improved direction $d^k \in \mathbb{R}^n$ is yielded by solving the following QP:

$$\text{QP}(x^k, H_k) \quad \begin{aligned} &\min \quad \nabla f(x^k)^T d + \frac{1}{2} d^T H_k d \\ &\text{s.t.} \quad g_i(x^k) + \nabla g_i(x^k)^T d \le 0, \quad \forall i \in I, \end{aligned} \tag{1.2}$$

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where H_k is an n-order symmetric positive definite matrix. Obviously, the subproblem above may be inconsistent since x^k is not necessary feasible. It is known that there exist at least four approaches which can be used to overcome this disadvantage, i.e., forcing the iteration x^k being feasible [5,6]; requiring suitable convexity on the constraint functions g_i [7]; norm-relaxed SQP technique [8,9] and perturbing the QP (1.2) by suitable artificial variables [10–12]. Then global convergence is followed by introducing a suitable line search. However, such line search may not obtain a full step of one even if the iteration points close to a solution of (1.1), that is the Maratos effect [13] may occur, and which is a necessary condition for superlinear convergence. One way to overcome this undesirable effect is to use a high-order correction direction which can be obtained by several techniques, such as solving a QP subproblem [14,15] or a system of linear equations [4,16,17], and so on.

Though the development of SQP methods, one can see that the global convergence and superlinear convergence of most SQP methods need assumption about the boundedness of iterative sequences which is difficult to ensure. Recently, based on the traditional QP (1.2), Solodov [7] proposed a new SQP algorithm for problem (1.2) with convex constraints without boundedness assumption on the iterative sequences. And the boundedness of iterative sequence can be obtained by introducing the idea of limiting infeasibility of iterative point, and a complete global convergence result can be obtained under mild assumptions. More recently, by combining the idea of Ref. [7] with the norm-relaxed SQP method, the authors [9] improved the algorithm in [7] such that the convexity is not required. However, the algorithms in [7,9] only enjoy global convergence property since there is no high-order correction direction to overcome the Maratos effect.

Building on the observations above, with a high-order correction direction by solving additional QP and an arc search, we aim to improve the algorithm in [7] such that it can achieve superlinear convergence, as a result, a new superlinearly convergent SQP method is presented.

The main features of the proposed algorithm are summarized as follows: • the main search direction is generated by solving one QP, and the correction direction is yielded by solving another QP; • the iterative sequence is proved to be bounded under reasonable assumptions; • the global convergence is proved under Slater CQ without Lipschitz-continuity request on the gradients of the constraint functions, and the superlinear convergence is obtained under suitable assumptions; • the proposed algorithm is further improved with a bidirectional line search technique, and the numerical results show that the proposed algorithm and its improvement are all promising.

2. Algorithm

Let $x^k \in \mathbb{R}^n$ be a current iteration point, similar to [7], the main search direction d^k is generated by solving the QP (1.2). To compute the next iteration point, we use the following l_1 merit function ψ_{β_k} which is similar to the one used in [7,18]:

$$\psi_{\beta_k}(x) = f(x) + \beta_k p(x), \quad \beta_k > 0, \tag{2.1}$$

where β_k is the current penalty parameter and the penalty function $p(x) = \sum_{i=1}^m g_i(x)_+$ with $g_i(x)_+ = \max\ \{0, g_i(x)\}$. By later analysis in Lemma 2.3, it is known, for suitable penalty parameter β_k, that d^k is an improved direction of both functions $\psi_{\beta_k}(x)$ and $p(x)$ at point x^k. Furthermore, for the sake of simplicity, we denote

$$\left.\begin{array}{ll} X = \{x \in \mathbb{R}^n : g_i(x) \le 0, \ \forall\, i \in I\}, & I_+^k \triangleq I_+(x^k) = \{i \in I : g_i(x^k) > 0\}, \\ D_k = \{d \in \mathbb{R}^n : g_i(x^k) + \nabla g_i(x^k)^T d \le 0, \ \forall\, i \in I\} & \end{array}\right\}. \tag{2.2}$$

In this work, the following assumption is assumed to be satisfied which is the basic assumption in [7].

Assumption A. (i) Functions f, g_i $(i \in I)$ are all continuously differentiable on $\mathbb{R}^n$, and g_i $(i \in I)$ are all convex functions; (ii) the feasible set X is bounded; and (iii) the strict interior of X is nonempty, i.e., there exists a point $\widehat{x}$ such that $g_i(\widehat{x}) < 0, \ \forall\, i \in I$.

The lemma below describes some important characters of the QP subproblem (1.2).

Lemma 2.1. *Suppose that Assumption A(i) holds and $X \neq \emptyset$. If the matrix H_k is symmetric positive definite, then*

(i) *$\bar{x} - x^k$ is a feasible solution of subproblem (1.2) with any $\bar{x} \in X$, and subproblem (1.2) has a unique solution d^k;*
(ii) *d^k is an optimal solution of (1.2) if and only if it is a KKT point; and*
(iii) *the optimal solution d^k of (1.2) equals zero if and only if x^k is a KKT point of (1.1).*

Proof. (i) Since $X \neq \emptyset$, we can choose a point $\bar{x} \in X$. So, combining the convexity and differentiability of g_i, we have $0 \ge g_i(\bar{x}) \ge g_i(x^k) + \nabla g_i(x^k)^T(\bar{x} - x^k), \ \forall\, i \in I$. This implies that $\bar{x} - x^k \in D_k$, that is the feasible set D_k of (1.2) is nonempty. Therefore, taking into account the positive definite property of H_k, one can conclude that the subproblem (1.2) has a unique solution, see [19, Corollary 3.4.2].

(ii) If d^k is a KKT point of (1.2), then it is an optimal solution since (1.2) is a convex programming. Conversely, if d^k is an optimal solution of (1.2), note that the Abadie CQ holds since the constraints of (1.2) are all linear, then d^k is a KKT point of (1.2).