Journal of Computational and Applied Mathematics 263 (2014) 152–159

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Option pricing using the fast Fourier transform under the double exponential jump model with stochastic volatility and stochastic intensity☆

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ARTICLE INFO

Article history:
Received 21 August 2013
Received in revised form 14 November 2013

Keywords:
Option pricing
Fast Fourier transform
Double exponential jump
Stochastic volatility
Stochastic intensity

ABSTRACT

This paper is based on the FFT (Fast Fourier Transform) approach for the valuation of options when the underlying asset follows the double exponential jump process with stochastic volatility and stochastic intensity. Our model captures three terms structure of stock prices, the market implied volatility smile, and jump behavior. Via the FFT method, numerical examples using European call options show effectiveness of the proposed model. Meanwhile, numerical results prove that the FFT approach is considerably correct, fast and competent.

1. Introduction

The Black–Scholes model and its extensions comprise one of the major developments in modern finance. The methodology for pricing options is developed by Black and Scholes [1]. Although the pricing in the Black–Scholes model can be done very accurately, these prices do not capture the volatility pattern observed from traded option prices. A number of papers are proposed for volatility dynamics, such as Hull and White [2], and Stein and Stein [3], but the most influential paper in the square root model is developed by Heston [4]. However, Heston's model has a serious disadvantage: the model does not consider the rare events such as financial crisis. To get more realistic models, Bates [5] proposes the addition of jump in his model. In Bates' framework, it is hard to disentangle the diffusion and jump risks since they are all driven by a single state variable, the diffusive volatility. Duffie, Pan and Singleton [6] synthesize and significantly extend the literature on affine asset pricing models by deriving a closed-form expression for an "extended transform" of an affine jump diffusion process, and highlight the impact on option 'smirks' of the joint distribution of jumps in volatility and jumps in the underlying asset price. Santa-Clara and Yan [7] find that the innovations to the two risks, respectively denoted by diffusion volatility process and jump intensity process in their model, have affected the expected return in the stock market. When they calibrate the model to the Standard & Poor's 500 index option prices from the beginning of 1996 to the end of 2002, they obtain time series of the implied diffusive volatility and jump intensity. The two components of risk vary substantially over time and show a high degree of persistence. In their model, it is the first model in which the jump intensity follows explicitly its own stochastic process. The empirical results show that the estimated correlation between the increments of the diffusive volatility and jump intensity is quite low. This is an evidence that the two processes are largely uncorrelated and do not support models

☆ This work is supported by the Fundamental Research Funds for the Central Universities (JBK130401).
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http://dx.doi.org/10.1016/j.cam.2013.12.009

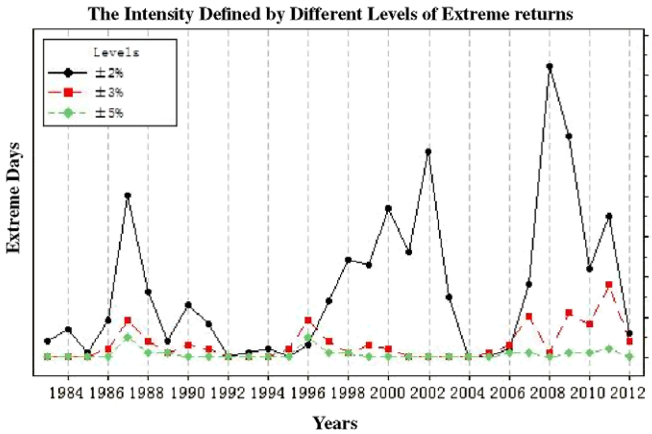

Fig. 1. The intensity defined by different levels of extreme returns.

that make jump intensity vary with the level of diffusive volatility. They also obtain the risk-adjusted dynamics of the stock, volatility, and jump intensity processes and use them to price European options.

Using stock return data of 10 years, Chang, Fuh and Lin [8] confirm the existence of switches in jump intensity and show that modeling the dynamic nature of jump dependence effectively captures overall changing volatility, supporting the use of this class of models to demonstrate stochastic volatility. Following the same way of Chang, Fuh and Lin [8], we analyze extreme returns of the Standard & Poor's 500 index from 1983 to 2013. Summary statistics are presented in Fig. 1 which shows the number of days for each year at different levels of returns exceeded, i.e., 2%, 3%, and 5%. We find that the jump intensity is consistently high or low and follows a similar mean-reverting process. Especially, in 2% threshold, jump intensity fluctuates around mean 30. Dates, empirically, demonstrate that the stochastic jump intensity model is better than the constant jump intensity model. Because of several previous studies and the display of the dates, we add the jump intensity which follows explicitly mean-reverting stochastic process to our model. An excellent contribution of our model is developing the model of Santa-Clara and Yan [7], which combines stochastic volatility, jump and mean-reverting jump intensity processes. Thus, our model better corresponds with the real market than the constant jump intensity model.

As for the distribution of the jump size, Merton [9] proposed the jump diffusion where the logarithm of jump size is normally distributed. Although Merton's model makes the asset pricing more realistic and explains the volatility smile in a stationary way, it cannot capture the leptokurtic feature of the return distribution. Another jump diffusion is studied by Kou [10], where the jump size has an asymmetric double exponential distribution. This double exponential jump diffusion model is able to reproduce the leptokurtic feature of the return distribution and the volatility smile observed in option prices. In addition, the empirical tests performed by Kou and Wang in [11] suggest that the double exponential jump diffusion model fits stock data better than the normal jump diffusion model. Because the double exponential distribution has both a high peak and heavy tails, it can be used to model both the overreaction (attributed to the heavy tails) and underreaction (attributed to the high peak) to outside news. Therefore, the double exponential jump diffusion is captured by our model, within stochastic volatility and stochastic intensity.

The finite difference method and the Monte Carlo simulation are usually used to value the options. But both ways are difficult to be applied in option pricing because they require substantially more computing time than the FFT approach. A large amount of the current literature on option valuation has fully applied Fourier analysis to determine option prices, such as Scott [12], and Bakshi and Chen [13]. But those papers do not consider the computational power of the FFT approach, which is one of the most fundamental advances in computing. Carr and Madan [14] describe an approach for numerically determining option values, which is designed to use the FFT approach to value options efficiently. Their use of the FFT approach in the inversion stage permits real-time pricing, marking, and hedging using realistic models, even for books with thousands of options. They anticipate that advantages of the FFT approach are generic to the widely known improvements in computation speed attained by this algorithm and is not connected to the particular characteristic function or process they chose to analyze. A number of recent papers use the FFT approach for a solution to the problem of option valuation (see e.g. Wong and Lo [15], Pillay and OHara [16], Zhang and Wang [17]).

A study closely related to this paper is that by Pillay and OHara [16], who present a general formula for valuing derivatives with mean reversion and stochastic volatility. Yet we focus on European options pricing by the FFT technique in the double exponential jumps setting with stochastic volatility and stochastic intensity. The model allows the covariance structure of the stocks to the intensity to be unrestricted, which proves to be important in the numerical analysis and empirical estimation. Numerical examples indicate that the formulas are easy to implement, and are accurate. Unfortunately, this approach has not been used for option pricing under double exponential jump process with stochastic volatility and stochastic intensity. Another contribution of our paper is employing the FFT technique to address our model and obtaining the solution of the characteristic function and the expression of the option pricing formula.