Journal of Computational and Applied Mathematics 259 (2014) 371–376

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Stochastic maximum principle for nonlinear optimal control problem of switching systems

Qurban Abushov [a], Charkaz Aghayeva [b,*,1]

[a] Institute of Cybernetics, National Academy of Science of Azerbaijan, Baku, Azerbaijan

[b] Industrial Engineering Department, Anadolu University, Eskisehir, Turkey

ARTICLE INFO

Article history:
Received 29 January 2013

Keywords:
Switching system
Nonlinear stochastic differential equations
Stochastic optimal control problem
Maximum principle
Adjoint stochastic differential equations
Switching law

ABSTRACT

The contribution of this paper is to present a stochastic maximum principle for an optimal control problem of switching systems. It presents necessary conditions of optimality for a broad class of switching systems, in which the dynamic of the constituent processes takes the form of stochastic differential equations. The restrictions on the transitions for the system are described through functional equality constraints on the end of each subsystem.

1. Introduction

A switching system where the dynamic behavior of interest can be described by a set of differential or difference equations, together with a set of rules for switching among these models, is a special type of hybrid system with the continuous law of movement. A change of the structure of the system means that at some moment it may go from one law of movement to another. After changing the structure, the characteristics of the initial condition of the system depends on its previous state. This situation joins them into a single system with variable structure. A broad class of these systems have stochastic behavior and have been modeled by the class of stochastic differential equations, see Arnold [1]; Boukas [2].

Optimization problems of switching systems have been recently attracting researchers from various fields in manufacturing, communication networks and traffic control such as Bengea [3]; Capuzzo [4]; Gao et al. [5]; Picolli [6]; Seidmann [7].

Stochastic control problems have a variety of practical applications in fields such as physics, biology, economics, management sciences, etc. The modern stochastic optimal control theory has been developed along the lines of Pontryagin's maximum principle and Bellman's dynamic programming [8,9]. The stochastic maximum principle has been first considered by Kushner [10]. Earliest results on the extension of Pontryagin's maximum principle to stochastic control problems are obtained in [11–14]. A general theory of stochastic maximum principle based on random convex analysis was given by Bismut [15]. Modern presentations of the stochastic maximum principle with backward stochastic differential equations are considered in [16]. By introducing a new approach, necessary and sufficient conditions of optimality are established in [17].

Switching optimal control is an approach to switching control, in which we seek strategies to minimize a cost function, or criterion of best performance. A classical approach for optimization and optimal control problems is to derive necessary conditions satisfied by an optimal solution.

* Corresponding author. Tel.: +90 5395601639.
E-mail address: c_aghayeva@anadolu.edu.tr (C. Aghayeva).

[1] The author published under the name Cherkez Agayeva until 2008.

0377-0427/$ – see front matter
http://dx.doi.org/10.1016/j.cam.2013.06.010

In this paper, backward stochastic differential equations have been used to establish a maximum principle for stochastic optimal control problems of switching systems. To our best acknowledge, such kinds of problems have been considered firstly by the authors in Agayeva and Abushov [18], Ch. Aghayeva and Q. Abushov [19], where the stochastic optimal control problem of switching systems with fixed points of variations are studied. The problem with unknown points without endpoint constraints is considered in [20]. The problems with a special type of endpoint constraints are derived in [21].

In this paper, the optimal control problem of stochastic switching systems with endpoint constraints is considered. We obtain a necessary condition of optimality in the form of a maximum principle for such systems, where the restrictions on transitions are described by equality constraints.

The rest of the paper is organized as follows. The notations, some basic definitions, the description of the main problem and the various assumptions used throughout this paper are given in Section 2. Section 3 is devoted to stochastic optimal control problem of a switching system without endpoint constraints. In this section we establish necessary condition of optimality for the case of uncontrolled diffusion. In Section 4, the maximum principle for the stochastic optimal control problem of a switching system with constraints at each switching mode is presented.

2. Preliminaries and formulation of the problem

Throughout this paper, we use the following notations. Let N be some positive constant, $\mathbb{R}^n$ denotes the n dimensional real vector space, $|\cdot|$ denotes the Euclidean norm in $\mathbb{R}^n$ and $\mathbb{E}$ represents the mathematical expectation. Assume that $w_t^1, w_t^2, \ldots, w_t^r$ are independent Wiener processes, which generate filtration $F_t^l = \bar{\sigma}(w_t^l, t_{l-1} \le t \le t_l),\ l = \overline{1,r},\ 0 = t_0 < t_1 < \cdots < t_r = T$. Let (Ω, F, P) be a probability space with filtration $\{F_t,\ t \in [0,T]\}$, where $F_t = \bar{\sigma}(F_t^l, l = \overline{1,r})$. $\mathbb{L}^2_{F^l}(a,b;\mathbb{R}^n)$ denotes the space of all predictable processes $x_t(\omega)$ such that $\mathbb{E}\int_a^b |x_t(\omega)|^2 dt < +\infty$. $\mathbb{R}^{m\times n}$ is the linear space of all linear transformations from $\mathbb{R}^m$ to $\mathbb{R}^n$. Let $O_l \subset \mathbb{R}^{n_l},\ Q_l \subset \mathbb{R}^{m_l},\ l = \overline{1,r}$, be open sets.

Consider the following stochastic control system with variable structure:

$$dx_t^l = g^l\left(x_t^l, u_t^l, t\right)dt + f^l\left(x_t^l, t\right)dw_t \quad t \in (t_{l-1}, t_l],\ l = \overline{1,r}, \tag{1}$$

$$x_{t_{l-1}}^l = \Phi^{l-1}\left(x_{t_{l-1}}^{l-1}, t_{l-1}\right) \quad l = \overline{2,r};\ x_{t_0}^1 = x_0, \tag{2}$$

$$u_t^l \in U_\partial^l \equiv \left\{u^l(\cdot,\cdot) \in \mathbb{L}^2_{F^l}\left(t_{l-1}, t_l; \mathbb{R}^{m_l}\right) | u^l(t,\cdot) \in U^l \subset \mathbb{R}^{m_l},\ l = \overline{1,r},\ \text{a.c.}\right\} \tag{3}$$

where $U^l,\ l = \overline{1,r}$ are non-empty bounded sets, and elements of U_∂^l are called admissible controls.

The problem is to find controls $u^1, u^2, \ldots, u^r$ and the switching law $(t_1, t_2, \ldots, t_r)$ such that the cost functional

$$J(u) = \sum_{l=1}^r \mathbb{E}\left[\varphi^l\left(x_{t_r}^l\right) + \int_{t_{l-1}}^{t_l} p^l\left(x_t^l, u_t^l, t\right)dt\right] \tag{4}$$

is minimized, where the functional $J(u)$ is determined on decisions of the system (1)–(3), which are generated by all admissible controls $U = U^1 \times U^2 \times \cdots \times U^r$ at conditions:

$$Eq^l\left(x_{t_l}^l\right) = 0, \quad l = \overline{1,r}. \tag{5}$$

Assume that the following requirements are satisfied:

I. Functions $g^l, f^l, p^l,\ l = \overline{1,r}$ and their derivatives are continuous in (x,u,t).

II. The derivatives of $g^l, f^l, p^l,\ l = \overline{1,r}$ are bounded by $N(1+|x|)$.

III. Functions $\varphi^l(x)\, l = \overline{1,r}$ are continuously differentiable and their derivatives are bounded by $N(1+|x|)$.

IV. Functions $\Phi^l(x,t) l = \overline{1,r-1}$ are continuously differentiable and their derivatives are bounded by $N(1+|x|)$.

V. Functions $q^l(x),\ l = \overline{1,r}$ are continuously differentiable and their derivatives are bounded by $N(1+|x|)$.

Consider the sets: $A_i = \mathrm{T}^{i+1} \times \prod_{j=1}^i O_j \times \prod_{j=1}^i Q_j,\ i = \overline{1,r}$, with the elements

$$\pi^i = (t_0, t_1, t_i, x_{t_0}^1, x_{t_2}^2, \ldots, x_{t_i}^i, u^1, u^2, \ldots, u^i).$$

Definition 1. The set of functions $\left\{x_t^l = x^l\left(t, \pi^l\right),\ t \in [t_{l-1}, t_l],\ l = \overline{1,r}\right\}$ is said to be a solution of Eqs. (1)–(2) with variable structure corresponding to an element $\pi^r \in \mathrm{A}_r$, if function $x_t^l \in O_l$ satisfies condition (2) on point t_l, while it is absolutely continuous on interval $[t_{l-1}, t_l]$ with probability 1 and satisfies the Eq. (1) almost everywhere.

Definition 2. The element $\pi^r \in A_r$ is said to be admissible if the pairs $\left(x_t^l,\ u_t^l\right),\ t \in [t_{l-1}, t_l],\ l = 1, \ldots, r$ are the solutions of system (1)–(3) which satisfy the conditions (5).

Definition 3. Let A_r^0 be the set of admissible elements. The element $\tilde{\pi}^r \in A_r^0$, is said to be an optimal solution of problem (1)–(5) if there exist admissible controls $\tilde{u}_t^l,\ t \in [t_{l-1}, t_l],\ l = \overline{1,r}$ and solutions $\left\{\tilde{x}_t^l,\ t \in [t_{l-1}, t_l],\ l = \overline{1,r}\right\}$ of system (1)–(2) such that pairs $\left(\tilde{x}_t^l, \tilde{u}_t^l\right),\ l = \overline{1,r}$ minimize the functional (4).