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On the single name CDS price under structural modeling

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ABSTRACT

Regulators, banks and other market participants realized that true assessment of the credit risk is more critical and complex than their ex-ante appraisals after the US Credit Crunch. They have turned their attention to complex credit risk models and credit instruments such as credit derivatives. Credit default swap contracts (CDSs) are the most common credit derivatives used for speculation and hedging purposes in the credit markets. Thus, in this paper we fundamentally study the pricing of a single name CDS via the discounted cash flow method with survival probability functions of two pioneer structural credit risk models, Merton model and Black–Cox model with constant barrier. Hence, this approach is not only a new one, but also provides a practical technique to price CDSs using publicly available data of equity returns.

1. Introduction

Credit risk is one of the most fundamental risks that financial and non-financial institutions are exposed to. Compared to other financial risks defined by Basel II (Basel Accord), credit risk has the biggest proportion. Therefore, credit risk management is very crucial for institutions. Besides risk management and hedging purposes, the USA Credit Crunch shows that integrated supervision and credit risk monitoring are also important in order to restrain potential credit losses. In retrospect, the lack of understanding of the credit and liquidity risks related to the financial instruments was one of the main reasons of the credit crisis started in the United States [1]. Due to their complex structures, risks related to credit derivatives were not evaluated accurately. Hence, credit derivatives are regarded as *scapegoat* of the credit crunch. However, if their structures are clearly understood and their risks are truly evaluated, use of credit derivatives would improve market depth and reduce information asymmetry of financial markets.

As a result of the globalization speed, the integration and interaction among credit markets have grown rapidly. To overcome competition and to get bigger slice of the pie in the market, new instruments like credit derivatives have been created. In order to liberate their capital for additional loan intermediation, banks and other lenders try to transfer their credit risk via direct loan selling, loan syndication, collateralized loan obligations (CLOs), and credit derivatives (credit default swaps CDSs) [2]. Therefore, assessment of the credit risk has become much harder due to complex structured credit instruments. In order to assess the credit risk, modern finance spends much effort to develop new models than ever. Furthermore, in 2004, Basel II Accord has encouraged financial institutions to develop and use internal credit risk models [3].

There are several approaches used to evaluate credit risks that are directly related to the estimation of the default probability for the credit applicant. Expert systems, neural networks, rating systems (including bank's internal rating systems) and credit scoring models are fundamental traditional credit assessment models (see [4–7] for further details).

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It is difficult to distinguish traditional and internal credit risk approaches, suggested by Basel II, since many reasonable ideas of traditional models are also used in the internal credit risk models. There are mainly two types of internal credit risk models: structural (firm value-based) and intensity-based models. In the setting of the structural models, the default is associated with the value of a firm's financial assets. Generally, the dynamics of the assets' value $V = \{V_t : 0 \leq t \leq T\}$ is given and the default event defined as a boundary condition on this process [3]. In other words, default time is defined as the time that the value of the assets breaches the (pre-determined) level L. Such a structural model was first introduced by Merton in 1974 [8]. Other structural models are extended versions of Merton's work, such as first-passage time models which eliminate some weaknesses of the Merton model.

On the other hand, intensity-based models focus directly on modeling the default probability. The basic idea which lies behind these models is that there is a possibility of default for an obligor at any instant time $t \in [0, T]$. The default is defined as the first jump of a counting process $N = \{N_t : 0 \leq t \leq T\}$ with intensity $\lambda = \{\lambda_t : 0 \leq t \leq T\}$. For details see [3,9], for instance.

Credit default swaps are the most widely used credit derivatives by financial and non-financial institutions. In general, there are three different approaches to price CDSs: hedge-based valuation, bond yield-based valuation, and discounted cash flow method. Hedge-based approaches depend on the fact that the price of the contracts with the same cash flows occurring at exactly the same time should be equal [10]. The bond yield-based valuation approaches assume that the CDS premium (risk premium) is the difference between price of risk free and risky bond [11]. On the other hand, discounted cash flow approaches rely on the equality of cash inflows and cash outflows. The present value of the expected CDS premium should then be equal to the present value of the expected default payment under the constant recovery rate assumption. The general formula that equates these two expectations is given by Beem [12] and Schoutens and Cariboni [3].

In this paper, we use the discounted cash flow method instead and obtain a closed form solution for a single name CDS under the Merton model (see Theorem 3.1). We assume that default can occur at payment dates or in the middle of two consecutive payment dates for simplicity. The default is then modeled as the first-passage of a constant default barrier. We also obtain the price of a single name CDS using the discounted cash flow method with the survival probability of the Black–Cox model (see Theorem 3.2), one of the earliest first-passage time models. Although the approach can be regarded as simple, it provides an appreciated closed form solution under both Merton and Black–Cox model with constant barrier. Further, the results can easily be implemented.

The remainder of the paper proceeds as follows. In Section 2, we give a brief review of the structural models and then explain the basic structure of the two pioneering structural models: Merton and Black–Cox models. Main results of this paper are presented in Section 3. It consists of CDS infrastructure and CDS pricing by combining the general equality of the cash inflows and cash outflows with survival probabilities of the Merton model and Black–Cox model with constant barrier. Section 4 presents a summary of contributions of the paper and conclusive remarks.

2. Structural models

This section is a short review of the literature on structural models and the derivations of their survival probabilities that will be used in the main contribution of this paper.

Structural models are models that are basically based on Black and Scholes [13] formulas for option pricing. The work of Merton [8] is one of the earliest of these models and it constructs the basis of credit risk models. According to Merton, stock holders can be considered as European call option holders. The option is written on the firm's assets with strike price equal to the value of only a single zero coupon bond which stands for the liability of a levered firm and can be paid at some certain maturity T. There are several structural models that follow Merton's study. The aims of these models are to improve this elegant but naive idea to tackle with the real economy problems. The article of Black and Cox [14] considers a single extension of the Merton model by adding early default possibility, modeled as a first-passage time. That is, the default time was modeled as the first time that the value of V breaks down the barrier L. In order to relax only one zero coupon bond assumption of the Merton model, Geske [15] proposed firm's liability as coupon bond in order to emphasize that there might be more than one liability. To loosen the constant interest rate assumption, however, the stochastic interest rate is introduced in the model and studied by Longstaff and Schwartz [16]. For more on structural and Lévy first-passage time models, we refer the reader to [3].

2.1. Merton model

The Merton model is the pioneer of the structural models and relies on the option pricing methodology. One of the basic assumptions of this model is that the asset values of a firm V can be described by a diffusion-type stochastic process following a geometric Brownian motion,

$$dV_t = \mu V_t dt + \sigma V_t dW_t. \tag{1}$$

The solution of this stochastic differential equation is

$$V_t = V_0 \exp\left\{\left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma W_t\right\},$$