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Abstract

The main objective of this paper is to study the fractional Fourier transform ($FrFT$) and its some basic properties. Applications of the $FrFT$ in solving generalized n^{th} order linear nonhomogeneous ordinary differential equations and a generalized wave equation are given. The generalized continuous wavelet transform and its inversion formula, and the Parseval relation using the fractional Fourier transform are also studied.

Keywords: fractional Fourier transform, generalized continuous wavelet transform, generalized wave equation, Fourier transform, mother wavelet

2000 MSC: 46F12, 26A33, 46A05, 47A30, 47G05

1. Introduction

Let $L^p(\mathbb{R})$ denotes the class of measurable functions of ϕ on $\mathbb{R}$ such that the integral $\int_{\mathbb{R}} |\phi(t)|^p\, dt$ is finite. Also let $L^\infty(\mathbb{R})$ be the collection of almost everywhere bounded functions. Hence endowed with norm:

$$\| \phi \|_p = \begin{cases} \left(\int_{\mathbb{R}} | \phi(t) |^p \, dt \right)^{1/p}, & 1 \le p < \infty, \\ ess \ \sup_{t\in\mathbb{R}} |\phi(t)|, & p = \infty. \end{cases}$$

The Fourier transform of a function $\phi \in L^1(\mathbb{R})$, is defined by

$$\hat{\phi}(\omega) = (\mathcal{F}\phi)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{-it\omega}\phi(t)dt, \quad \omega \in \mathbb{R}, \tag{1}$$

and if $\hat{\phi} \in L^1(\mathbb{R})$, then the inverse Fourier transform is given by

$$\phi(t) = (\mathcal{F}^{-1}\hat{\phi})(t) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{it\omega}\hat{\phi}(\omega)d\omega, \quad t \in \mathbb{R}. \tag{2}$$

The $FrFT$ is a generalization of the classical Fourier transform which depends on an angle θ, has many applications in several areas, including communications, optics, quantum physics, signal and image processing, etc. The one dimensional $FrFT$ $[1-5]$ with an angle θ of $\phi(t)$ denoted by $(\mathcal{F}^\theta\phi)(\omega) = \hat{\phi}^\theta(\omega)$ is given by

$$(\mathcal{F}^\theta\phi)(\omega) = \hat{\phi}^\theta(\omega) = \int_{\mathbb{R}} K^\theta(t,\omega)\phi(t)dt, \tag{3}$$

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