Accepted Manuscript

An hybrid multigrid method for convection–diffusion problems

S. Chaabane Khelifi, N. Méchitoua, F. Hülsemann, F. Magoulès

PII: S0377-0427(13)00259-8
DOI: http://dx.doi.org/10.1016/j.cam.2013.05.003
Reference: CAM 9159

To appear in: *Journal of Computational and Applied Mathematics*

Received date: 2 January 2013
Revised date: 12 April 2013

Please cite this article as: S. Chaabane Khelifi, N. Méchitoua, F. Hülsemann, F. Magoulès, An hybrid multigrid method for convection–diffusion problems, *Journal of Computational and Applied Mathematics* (2013), http://dx.doi.org/10.1016/j.cam.2013.05.003

An hybrid multigrid method for convection-diffusion problems

S. Chaabane Khelifi[a,c,*], N. Méchitoua[a], F. Hülsemann[b], F. Magoulès[c]

[a]EDF R&D, MFEE, 6 Quai Wattier F-78400 Chatou
[b]EDF R&D, SINETICS, 1 Avenue du général de gaulle, F-92140 Clamart
[c]Ecole Centrale de Paris, Grande voie des Vignes, F-92295 Chatenay-Malabry

Abstract

This paper presents an advanced performance study of a multigrid method designed for convection diffusion problems developed in [21]. The proposed scheme with the separation of the operators enables an individual treatment for each operator: while the piecewise constant operator is used for the convective part, each off-diagonal entry of the coarse diffusion operator is scaled by a geometric factor. Numerical examples illustrate the fast convergence and the outstanding robustness of the proposed method, compared to other known methods.

Keywords: Multigrid, convection-diffusion, aggregation, rescaling, computational fluid dynamics

1. Introduction

During the simulation of some physical phenomena, typically in the fluid dynamics field (CFD), the solution of large linear systems is usually required. With the ongoing increase of the complexity of the problems to treat, the solution phase may be very costly. It is not sufficient to use the latest technology of computers. An effort should be put into algorithms for solving such systems.

Multigrid methods (MG) are known to be a viable alternative to many other solution strategies especially for elliptic dominated problems. They are the fastest numerical methods for solving elliptic boundary value problems [20]. The key ingredient of the success of the MG method is the interplay between the smoothing procedure and the coarse grid correction. At the end of the smoothing process, the error is usually smooth, i.e. the oscillatory components of the error are eliminated. The coarse grid correction is based on the use of smaller problems. Once the transfer of information to a coarser grid, through a restriction operator, is done, a coarse grid system is solved. The solution found on the coarse grid is then transferred back to the fine grid through interpolation and used to correct the first approximation. The complementarity between the two processes lies in the fact that for each error component, there exists a grid on which this component is oscillatory so it can be effectively reduced by smoothing.

*Corresponding author

Email addresses: sanachaabane@yahoo.fr (S. Chaabane Khelifi), namane.mechitoua@edf.fr (N. Méchitoua), frank.hulsemann@edf.fr (F. Hülsemann), frederic.magoules@hotmail.com (F. Magoulès)