Journal of Computational and Applied Mathematics 259 (2014) 730–740

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

DRBEM solution of mixed convection flow of nanofluids in enclosures with moving walls

S. Gümgüm [a,*], M. Tezer-Sezgin [b,1]

[a] Department of Mathematics, İzmir University of Economics, 35330, İzmir, Turkey

[b] Department of Mathematics, Institute of Applied Mathematics, Middle East Technical University, 06531, Ankara, Turkey

ARTICLE INFO

Article history:
Received 23 December 2012
Received in revised form 27 March 2013

Keywords:
Nanofluids
DRBEM
Mixed convection
Lid-driven
Heat source
BEM

ABSTRACT

This paper presents the results of a numerical study on unsteady mixed convection flow of nanofluids in lid-driven enclosures filled with aluminum oxide and copper–water based nanofluids. The governing equations are solved by the Dual Reciprocity Boundary Element Method (DRBEM), and the time derivatives are discretized using the implicit central difference scheme. All the convective terms and the vorticity boundary conditions are evaluated in terms of the DRBEM coordinate matrix. Linear boundary elements and quadratic radial basis functions are used for the discretization of the boundary and approximation of inhomogeneity, respectively. Solutions are obtained for several values of volume fraction (φ), the Richardson number (Ri), heat source length (B), and the Reynolds number (Re). It is disclosed that the average Nusselt number increases with the increase in volume fraction, and decreases with an increase in both the Richardson number and heat source length.

1. Introduction

Nanofluids are the mixture of nano-sized particles suspended in a base fluid. Base fluids, such as water, engine oil and ethylene glycol have low heat transfer performance. Therefore, various techniques are applied to enhance the heat transfer of these fluids. One of them is the use of solid particles as an additive suspended into the base fluid. The improved heat transfer performance of nanofluids is due to the fact that dispersing high thermal conductivity nanoparticles in a base fluid increases the thermal conductivity of such mixtures, and enhances their overall heat transfer capability. Mixed convection is an important heat transfer mechanism and has applications in electronic cooling, drying, heat exchangers and insulation of buildings. It is the combination of forced and natural convection. Thus, the effects of both natural and forced convection influence the governing equations [1].

There are a number of recent studies on the mixed convection flow of nanofluids in cavities. Tiwari and Das [2] analyzed the heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids with finite volume approach using the SIMPLER algorithm. They found that both the Richardson number and the direction of the moving walls affect the fluid flow and heat transfer in the cavity. In another study, Talebi et al. [3] investigated the laminar mixed convection flows through a copper–water nanofluid in a square lid-driven cavity. They used the finite volume method for the numerical solution, and found that at the fixed Reynolds number, the solid concentration affects the flow pattern and thermal behavior particularly for a higher Rayleigh number. Mahmoodi [4] analyzed the mixed convection fluid flow and heat transfer in lid-driven enclosures filled with the Al_2O_3–water nanofluid numerically using the finite volume method

* Corresponding author. Tel.: +90 232 4888307; fax: +90 232 2792626.
E-mail addresses: sevin.gumgum@ieu.edu.tr, gumgumsevin@hotmail.com (S. Gümgüm), munt@metu.edu.tr (M. Tezer-Sezgin).
1 Tel.: +90 312 2105376; fax: +90 312 2102972.

0377-0427/$ – see front matter
http://dx.doi.org/10.1016/j.cam.2013.05.006

with SIMPLER algorithm. The results show that at low Richardson numbers, a primary counter-clockwise vortex is formed inside the enclosure. Numerical simulation of mixed convection flows in a square lid-driven cavity partially heated from below using nanofluids is studied by Mansour et al. [5]. The finite difference method (FDM) was employed to solve the dimensionless governing equations of the problem. They observed that increasing solid volume fraction leads to decrease in both the activity of the fluid motion and fluid temperature, however, it leads to increase in the corresponding average Nusselt number. In a recent study, Rahman et al. [6] investigated the behavior of nanofluids in an inclined lid-driven triangular enclosure by using the Galerkin finite element method (FEM). They observed that solid volume fraction strongly influenced the fluid flow and heat transfer in the enclosure at the three convective regimes.

In the literature, mixed convection flow of nanofluids in enclosures are simulated using numerical methods which discretize the whole domain of the problem such as FVM, FDM and FEM. Thus, the resulting system of algebraic equations is very large in size due to the large number of nodal points in the region which has to be taken to achieve a good accuracy. On the other hand, the boundary element method discretizes only the boundary of the region reducing the size of the resulting systems. But, a domain integral results in BEM due to the inhomogeneity when the equation is Poisson's type. This causes loss of boundary only nature of BEM. The DRBEM handles this problem by transforming the domain integral to a boundary integral. The DRBEM has also the flexibility of using fundamental solution of Laplace equation which is the main differential operator in mixed convection flow. In DRBEM all the convective terms and derivative type boundary conditions are approximated using coordinate matrix in terms of radial basis functions. These are the main advantages of DRBEM compared to all other domain discretization numerical methods. The application of DRBEM for solving natural convection flow of nanofluids is given by Gümgüm and Tezer-Sezgin [7] which uses FDM—in time and DRBEM—in space domains. The results are provided for Ra values up to 10^6. DRBEM application is extended to solve also the natural convection flow of micropolar fluids by the same authors [8].

In this paper, DRBEM formulation is given for solving mixed convection flow of nanofluid equations in terms of stream function, vorticity and temperature by using the fundamental solution of Laplace equation, and keeping all the other terms as inhomogeneity [9]. The DRBEM reduces all calculations to the evaluation of the boundary integrals discretizing only the boundary of the region. The unknown vorticity boundary conditions and all the spatial derivatives are easily obtained by using coordinate matrix which contains only radial basis functions. DRBEM application of unsteady mixed convection flow of nanofluids gives rise to systems of initial value problems in time which are approximated by implicit Euler scheme. Considerably small number of boundary elements are used resulting in small sized systems to be solved compared to all the other domain discretization methods. All the original unknowns (stream function, vorticity and temperature) are obtained at all transient levels including steady-state at a cheap expense due to the boundary nature of DRBEM. To the best of author's knowledge this is the first application of DRBEM for solving mixed convection flow of nanofluids.

2. Mathematical formulation

The non-dimensional unsteady momentum and energy equations for nanofluids can be written in terms of stream function (ψ), vorticity (ω) and temperature (T), as [4]

$$\begin{aligned}
&\nabla^2\psi = -\omega \\
&\frac{1}{Re}\frac{\mu_{eff}}{\rho_{nf}\nu_f}\nabla^2\omega = \frac{\partial\omega}{\partial t} + u\frac{\partial\omega}{\partial x} + v\frac{\partial\omega}{\partial y} - Ri\frac{(\rho\beta)_{nf}}{\rho_{nf}\beta_f}\frac{\partial T}{\partial x} \\
&\frac{1}{PrRe}\frac{\alpha_{nf}}{\alpha_f}\nabla^2 T = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}
\end{aligned} \tag{1}$$

where $(x, y) \in \Omega \subset R^2$, $t > 0$. Ri, Re and Pr are the Richardson, Reynolds and Prandtl numbers, respectively. $\nu_f = \mu_f/\rho_f$ is the kinematic viscosity of the fluid. μ_f and ρ_f are the dynamic viscosity and the density of the fluid. The velocities are given in terms of stream function as $u = \partial\psi/\partial y$, $v = -\partial\psi/\partial x$ and the vorticity is defined by $\omega = \partial v/\partial x - \partial u/\partial y$.

The density ρ_{nf}, the heat capacitance $(\rho C_p)_{nf}$ and the thermal expansion coefficient $(\rho\beta)_{nf}$ of the nanofluid are defined as $\rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_s$, $(\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_s$, and $(\rho\beta)_{nf} = (1-\varphi)(\rho\beta)_f + \varphi(\rho\beta)_s$, respectively [4,10]. The effective dynamic viscosity of the nanofluid is taken in the first problem as in [4,11], $\mu_{eff} = \mu_f(1+7.3\varphi+123\varphi^2)$, and in the second problem as in [5], $\mu_{eff} = \mu_f/(1-\varphi)^{2.5}$. The effective thermal conductivity of the nanofluid is approximated by the Maxwell–Garnett's model [12], $\kappa_{eff} = \left[\frac{\kappa_s+2\kappa_f-2\varphi(\kappa_f-\kappa_s)}{\kappa_s+2\kappa_f+\varphi(\kappa_f-\kappa_s)}\right]\kappa_f$. The use of this equation is restricted to spherical nanoparticles where it does not account for other shapes of nanoparticles. This model is found to be appropriate for studying heat transfer enhancement using nanofluids [13]. The thermal diffusivity of the nanofluid is given as [4], $\alpha_{nf} = \kappa_{eff}/(\rho C_p)_{nf}$. φ is nanoparticle volume fraction. eff, nf, s and f refer to effective, nanofluid, solid and fluid, respectively. ($\varphi = 0$ refers to pure base fluid and $0 < \varphi \leq 0.2$ refers to nanofluid.)

The equations in (1) are supplied with the appropriate initial and boundary conditions according to the physics of the mixed convective flow of nanofluids in lid-driven enclosures. The fluid in the cavity is a water-based nanofluid containing aluminum oxide (Al_2O_3) and copper (Cu) nanoparticles. It is assumed that the base fluid and nanoparticles are in thermal