Journal of Computational and Applied Mathematics 259 (2014) 752–759

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

A multidimensional characteristic-based method for making incompressible flow calculations on unstructured grids

M.Y. Hashemi [a,*], K. Zamzamian [b]

[a] Department of Mechanical Engineering, Azarbaijan Shahid Madani University, Tabriz, Iran

[b] Department of Mechanical Engineering, Tabriz branch, Islamic Azad University, Tabriz, Iran

ARTICLE INFO

Article history:
Received 11 February 2013
Received in revised form 1 May 2013

Keywords:
Artificial compressibility
Multidimensional characteristic
Unstructured grid
Navier–Stokes equations

ABSTRACT

This paper presents a new multidimensional characteristic-based scheme (*MCB*) for numerical simulation of incompressible flows on unstructured grids in conjunction with the artificial compressibility method. The original *MCB* scheme has been proposed by the authors already and this research is the extension of the aforementioned scheme to unstructured grids. The significant difference between *MCB* and the conventional characteristic-based scheme (*CB*) is the multidimensional nature of the scheme, which allows information to propagate in any direction instead of only normal to the cell interface. In addition, local time stepping and the residual smoothing technique have been used for convergence acceleration. The accuracy and utility of the proposed scheme have been studied by means of numerical tests for different Reynolds numbers and the results obtained using the new scheme are in good agreement with the standard benchmark solution in the literature.

1. Introduction

The main problem of computational fluid dynamics (CFD) for configurations with complex geometry is mesh generation. The structured grid methods have a disadvantage for complex geometries [1]. The main advantage of the unstructured grid methods is the facility of grid generation for complex configurations [2]. However, the computational costs and memory requirements are generally higher than for their structured grid counterparts.

The original characteristic-based method (*CB*) for the artificial compressibility approach [3] for solving the incompressible flow equations was proposed by Drikakis et al. [4]. Then the revised *CB* method was used on unstructured grids for incompressible flow solutions by X. Su et al. [5]. The multidimensional upwind characteristic-based (*MCB*) method was proposed by Zamzamian and Razavi [6] for solving the incompressible flow equations on a structured grid; it is based on the artificial compressibility method. They showed that the *MCB* scheme is robust and powerful for modeling incompressible viscous flows and for achieving high accuracy and remarkable advantages in convergence rate with respect to the conventional *CB* scheme. The main objective of the present paper is to use the efficient *MCB* method on an unstructured grid.

2. The governing equations

The Navier–Stokes equations for two-dimensional incompressible flows modified by artificial compressibility can be expressed as

$$\oint_{\Omega} \frac{\partial \mathbf{W}}{\partial t} dV + \oint_{C} (\mathbf{F}^I dS_x + \mathbf{G}^I dS_y) = \frac{1}{Re} \oint_{C} (\mathbf{F}^V dS_x + \mathbf{G}^V dS_y) \tag{1}$$

* Corresponding author. Tel.: +98 9127044971.
E-mail address: m.y.hashemi@azaruniv.ac.ir (M.Y. Hashemi).

0377-0427/$ – see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.cam.2013.06.008

where

$$\mathbf{W} = \begin{bmatrix} p \\ u \\ v \end{bmatrix}, \qquad \mathbf{F}^I = \begin{bmatrix} \beta u \\ u^2 + p \\ uv \end{bmatrix}, \qquad \mathbf{G}^I = \begin{bmatrix} \beta v \\ vu \\ v^2 + p \end{bmatrix},$$
$$\mathbf{F}^V = \begin{bmatrix} 0 \\ \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial x} \end{bmatrix}, \qquad \mathbf{G}^V = \begin{bmatrix} 0 \\ \dfrac{\partial v}{\partial x} \\ \dfrac{\partial v}{\partial y} \end{bmatrix}. \tag{2}$$

Here $\mathbf{W}$ is the vector of primitive variables, and $\mathbf{F}^I$, $\mathbf{G}^I$ and $\mathbf{F}^V$, $\mathbf{G}^V$ are convective and viscous flux vectors, respectively. The artificial compressibility parameter and Reynolds number are shown as β and Re, respectively. The above equations have been nondimensionalized on the basis of the following scalings:

$$x = \frac{x^*}{l^*}, \qquad y = \frac{y^*}{l^*}, \qquad t = \frac{t^*}{l^*/U_{ref}},$$
$$u = \frac{u^*}{U_{ref}}, \qquad v = \frac{v^*}{U_{ref}}, \qquad p = \frac{p^* - p_{ref}}{\rho_{ref} U_{ref}^2}. \tag{3}$$

The discretized form of Eqs. (1) at cell i is obtained:

$$A_i \frac{\partial \mathbf{W}_i}{\partial t} + \sum_{j=1}^{m} \mathbf{F}_j^I (\Delta S_x)_j + \sum_{j=1}^{m} \mathbf{G}_j^I (\Delta S_y)_j = \frac{1}{Re} \left[\sum_{j=1}^{m} \mathbf{F}_j^V (\Delta S_x)_j + \sum_{j=1}^{m} \mathbf{G}_j^V (\Delta S_y)_j \right] \tag{4}$$

where A_i is the cell area and m is the number of edges for any cells. Examples of computational unstructured grids that are used for finite-volume *MCB* flow solvers are shown in Fig. 3.

3. The solution algorithm

3.1. The two-dimensional characteristic structure for artificial compressibility equations

To derive the characteristic relations of incompressible flows, their corresponding "Euler equations" are considered [7]. These equations modified by artificial compressibility for deriving two-dimensional characteristic structures are

$$\begin{aligned} &\frac{\partial p}{\partial t} + \beta \frac{\partial u}{\partial x} + \beta \frac{\partial v}{\partial y} = 0 \\ &\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + u \frac{\partial u}{\partial y} + \frac{\partial p}{\partial x} = 0 \\ &\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + u \frac{\partial v}{\partial y} + \frac{\partial p}{\partial y} = 0. \end{aligned} \tag{5}$$

To obtain the characteristic structure of equations, a characteristic surface in the form of $f(x, y, t) = 0$ is assumed. Using the kinematics relations for relating the partial derivatives to exact derivatives corresponding to the assumed surface, one gets the following system of equations [8,9]:

$$\begin{bmatrix} \dfrac{f_t}{\beta} & f_x & f_y \\ f_x & \psi & 0 \\ f_y & 0 & \psi \end{bmatrix} \begin{bmatrix} dp \\ du \\ dv \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

where the subscripts stand for the partial differentiation and ψ is defined as

$$\psi = \frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} + v \frac{\partial f}{\partial y}. \tag{7}$$

For compatibility requirements of Eqs. (6), the determinant of the coefficient matrix is set to zero; hence,

$$\psi = 0, \qquad \psi = \frac{\beta}{f_t} \left(f_x^2 + f_y^2 \right). \tag{8}$$