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A convex optimization method to solve a filter design problem[☆]

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ABSTRACT

In this paper, we formulate the feasibility problem corresponding to a filter design problem as a convex optimization problem. Combined with a bisection rule, this leads to an algorithm for minimizing the design parameter in the filter design problem. A safety margin is introduced to solve the numerical difficulties when solving this type of problem numerically. Numerical experiments illustrate the validity of this approach for larger degrees of the filter as compared to similar previous algorithms.

1. Motivation

In this paper, a low-pass filter design problem is considered in which the parameters of the problem are the passband ripple, the stopband attenuation, and the degree of the filter transfer function. In [1], Genin et al. show that such a filter design problem can be solved by combining a bisection rule on one of these design parameters and an optimization scheme solving a feasibility problem. The feasibility problem can be reformulated as a convex optimization problem over the cone of nonnegative trigonometric polynomials. In [2–5], it is shown that these polynomials can be parameterized by using Hankel and/or Toeplitz matrices. In [1], the authors focus on minimizing one of the three parameters, namely, the stopband attenuation. Their optimization method is based on the dual formulation and is implemented by using the LMI Toolbox of MATLAB, but it breaks down as soon as the degree of the filter is greater than 7.

The aim of this paper is to design an algorithm that can be used for much higher degrees of the filter. We first reformulate the feasibility problem of the filter design problem as a convex optimization one, more precisely, as a conic programming problem [6,7]. This approach is then used to develop an algorithm to solve the filter design problem in which the passband ripple and the stopband attenuation are minimized simultaneously. In contrast to the approach in [1], our algorithm solves the primal formulation of the problem. We also introduce a safety margin to make the results of the algorithm more accurate. In our numerical experiments, we show that the primal convex feasibility problem can be solved efficiently and accurately by using the `cvx` Toolbox [8,7,9]. This allows us to take much larger values of the filter degree. Finally, we discuss the behavior of the minimal value of the whole problem when the filter degree, the passband, and the difference between the stopband and the passband change.

2. Nonnegative trigonometric polynomials

In this section, we introduce some results on complex-valued functional systems and on the cones of nonnegative trigonometric polynomials on the unit circle and its symmetric arcs. One can find more details in [4] or [10, Sections 2.1–2.3].

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2.1. Complex-valued functional systems in the complex plane

Let Γ be an arbitrary set in the complex plane. Given a system $\mathcal{S} = \{\psi_0(z), \ldots, \psi_{r-1}(z)\}$ of linearly independent complex-valued functions $\psi_i(z)$ on Γ and a real-valued weight function $\phi(z)$ that is nonnegative on Γ, define the finite-dimensional cone

$$\mathcal{K} = \left\{ P(z) : P(z) = \phi(z) \sum_{i=0}^{N-1} Q_i(z) Q_i(z)^*, Q_i(z) \in \mathcal{F}(\mathcal{S}),\ i = 0, 1, \ldots, N-1 \right\}, \tag{1}$$

where

$$\mathcal{F}(\mathcal{S}) = \left\{ Q(z) : Q(z) = \sum_{k=0}^{r-1} Q_k \psi_k(z), Q_k \in \mathbb{C},\ k = 0, 1, \ldots, r-1 \right\} \tag{2}$$

and M^* denotes the complex conjugate (transposed matrix) of (a matrix) M. We take $N \geq r$. Define the squared functional system

$$\mathcal{S}^2 = \{v_{ij}(z) = \phi(z)\psi_i(z)\psi_j(z)^*,\ i, j = 0, 1, \ldots, r-1\}$$

and two vector functions, $\psi(z) = [\psi_0(z), \ldots, \psi_{r-1}(z)]^T$, and $v(z) = [v_0(z), \ldots, v_{s-1}(z)]^T$, whose components span a finite-dimensional space that covers $\mathcal{S}^2$.

Continuing, define two spaces:

$$E = \{P = (P_0, \ldots, P_{s-1}) : P_k \in \mathbb{C}, k = 0, 1, \ldots, s-1\},$$
$$F = \{W = (W_{ij}) : W_{ij} = W_{ji}^* \in \mathbb{C}, i, j = 0, 1, \ldots, r-1\},$$

with corresponding complex-valued inner products, respectively,

$$\langle ., . \rangle_E : E \times E \to \mathbb{C}, (P, Q) \mapsto \langle P, Q \rangle_E = \frac{1}{2} \sum_{i=0}^{s-1} (Q_i^* P_i + Q_i P_i^*),$$

$$\langle ., . \rangle_F : F \times F \to \mathbb{C}, (X, Y) \mapsto \langle X, Y \rangle_E = \sum_{i,j=0}^{r-1} Y_{ij}^* X_{ij}.$$

We can define a linear operator $\Lambda : E \to F$ as

$$\Lambda(P) = \frac{1}{2} \sum_{k=0}^{s-1} (P_k \Lambda_k + P_k^* \Lambda_k^*),$$

where $\{\Lambda_k\}_k \subset F$ is given such that $\Lambda(v(z)) = \phi(z)\psi(z)\psi(z)^*, \forall z \in \Gamma$.

2.2. Nonnegative trigonometric polynomials

We summarize the results given in [10, Section 2.3]. A trigonometric polynomial of degree d is of the form

$$p(e^{j\theta}) = \sum_{k=0}^{d} \Big(a_k \cos(k\theta) + b_k \sin(k\theta) \Big), \quad \theta \in (-\pi, \pi] \tag{3}$$

with $a_k, b_k \in \mathbb{R},\ k = 0, 1, \ldots, d$. Without loss of generality, we can assume that $b_0 = 0$. If we set $z = e^{j\theta}$, where $j = \sqrt{-1}$, then z belongs to the unit circle $\mathbb{T}$, and it is easy to see that

$$p(z) = \frac{1}{2} \left(\sum_{k=0}^{d} (a_k + jb_k) z^{-k} + \sum_{k=0}^{d} (a_k - jb_k) z^k \right).$$

If $b_k = 0, k = 0, 1, \ldots, d$, we call p a cosine polynomial.

Let ω_z denote the argument of the complex number z with $\omega_z \in (-\pi, \pi]$. For each pair $(u, v) \in \mathbb{T}^2$ with $0 \leq \omega_v - \omega_u < 2\pi$, let us denote the arc from u to v on $\mathbb{T}$ as

$$\mathbb{T}_{uv} = \{z \in \mathbb{T} : \omega_u \leq \omega_z \leq \omega_v\}.$$

When $0 \leq \omega_u < \pi$, we write $\mathbb{T}_u, \mathbb{T}_{\bar{u}}$ instead of $\mathbb{T}_{\bar{u}u}, \mathbb{T}_{u\bar{u}}$, respectively.

As a particular case in the previous subsection, we take $\mathcal{S} = \{1, z, \ldots, z^d\}$, $\phi(z) = 1$, and Γ as one of the subsets $\mathbb{T}$ or $\mathbb{T}_{uv}$. Then $\psi(z)$ can be chosen as $\psi(z) = [1, z, \ldots, z^d]^T$, and a minimal basis for $\mathcal{S}^2$ corresponds to the vector function $v(z) = [1, z, \ldots, z^{2d}]^T$. This implies that $r = d + 1$ and $s = 2d + 1$. Additionally, $E = \mathbb{C}^{d+1}$ and $F = \mathbb{H}^{d+1}$, the set of