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On a perturbed Sparre Andersen risk model with threshold dividend strategy and dependence

Zhimin Zhang *

College of Mathematics and Statistics, Chongqing University, Chongqing, 401331, PR China

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ABSTRACT

In this paper, we consider a Sparre Andersen risk model perturbed by a Brownian motion, where the individual claim sizes are dependent on the interclaim times. We assume that dividends are paid off under a threshold strategy. Integral and integro-differential equations satisfied by the Gerber–Shiu functions are obtained, and a solution procedure is also proposed.

1. Introduction

Consider the following Sparre Andersen risk model that is perturbed by a Brownian motion,

$$U_t^{\infty} = u + ct - \sum_{i=1}^{N_t} Y_i + \sigma B_t, \tag{1.1}$$

where $u \geq 0$ is the initial surplus and $c > 0$ is the premium rate. The claim number process $\{N_t, t \geq 0\}$, defined by $N_t = \max\{i : V_1 + V_2 + \cdots + V_i \leq t\}$, is a renewal process, where V_i for $i \geq 2$ denotes the interclaim time between the $(i-1)$th and the ith claim arrival and V_1 is the time until the first claim arrival. $\{Y_i, i \geq 1\}$ is a sequence of strictly positive random variables representing the individual claim amounts. Finally, $\{B_t, t \geq 0\}$, independent of other stochastic quantities, is a standard Brownian motion, and $\sigma > 0$ is the diffusion volatility.

Recently, risk models with dependence between interclaim times and individual claim sizes have drawn a lot of attention in ruin theory; see e.g. [1–8] and references therein. We find that most attention is focused on the model without diffusion perturbation, however, such model cannot capture small fluctuations in the surplus process. Zhang and Yang [9] considered a perturbed risk model with Farlie–Gumbel–Morgenstern (FGM) copula dependence structure. Zhang et al. [10] proposed a perturbed Sparre Andersen risk model with a more general dependency structure. Note that this dependence includes that of [9] as a special case. In this paper, we will apply the tractable structure proposed in [10] to model the dependence. Assume that $\{(V_i, Y_i), i \geq 1\}$ form a sequence of i.i.d. random vectors distributed like a canonical random vector (V, Y) with joint probability density function $f_{V,Y}$ of the following form

$$f_{V,Y}(t, y) = \sum_{i=1}^{n} e^{-\lambda_i t} f_i(y), \tag{1.2}$$

* Tel.: +86 13527478239.
E-mail addresses: zmzhang@cqu.edu.cn, cquzzm@163.com.

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where n is a strictly positive integer, $\lambda_i > 0$ for $i = 1, 2, \ldots, n$, and f_i's are continuous functions defined on $(0, \infty)$. As long as $f_{V,Y}(t, y)$ is nonnegative, some of f_i's can take negative values. We remark that the form in (1.2) includes the following dependency structures studied in the literature.

- (Dependence model by Boudreault et al. [2]). In order to model natural catastrophic events such as earthquakes, [2] proposed the conditional p.d.f. of the claim sizes given interclaim times as

$$f_{Y|V}(y|t) = e^{-\beta t} q_1(y) + (1 - e^{-\beta t}) q_2(y), \quad t > 0,\ y > 0,\ \beta > 0, \tag{1.3}$$

  where q_1, q_2 are two p.d.f.'s. Assume that marginal interclaim time density is exponential with parameter $\lambda > 0$. We can recover this dependency model with $n = 2$ in (1.2) by setting $\lambda_1 = \lambda + \beta$, $\lambda_2 = \lambda$, $f_1(y) = \lambda(q_1(y) - q_2(y))$, $f_2(y) = \lambda q_2(y)$.
- (The FGM copula with exponential marginals) Bargès et al. [11] studied the capital allocation problems for insurance company with several business lines. They applied FGM copula with exponential distributed risks to model the business lines. For the bivariate case, it is assumed that

$$f_{V,Y}(t, y) = (1 + \theta)\beta_1 e^{-\beta_1 t} \beta_2 e^{-\beta_2 y} - \theta 2\beta_1 e^{-2\beta_1 t} \beta_2 e^{-\beta_2 y} - \theta \beta_1 e^{-\beta_1 t} 2\beta_2 e^{-2\beta_2 y} + \theta 2\beta_1 e^{-2\beta_1 t} 2\beta_2 e^{-2\beta_2 y}, \tag{1.4}$$

  where $\theta \in [-1, 1]$, $\beta_1, \beta_2 > 0$. In this case, we can write $f_{V,Y}$ in the form of (1.2) with $n = 2$, $\lambda_1 = \beta_1$, $\lambda_2 = 2\beta_1$, $f_1(y) = (1 + \theta)\beta_1\beta_2 e^{-\beta_2 y} - 2\theta\beta_1\beta_2 e^{-2\beta_2 y}$, $f_2(y) = 4\theta\beta_1\beta_2 e^{-2\beta_2 y} - 2\theta\beta_1\beta_2 e^{-\beta_2 y}$.
- (The generalized FGM copula in the compound Poisson risk model). Cossette et al. [3] applied the generalized FGM copula to describe the dependency structure. Let

$$h_1(u) = u^a (1 - u)^b, \qquad h_2(v) = v^c (1 - v)^d,$$

  where a, b, c, d are integers larger than 1. The bivariate p.d.f. is given by

$$f_{V,Y}(t, y) = f_V(t) f_Y(y) + \theta h_1'(F_Y(y)) h_2'(F_V(t)) f_V(t) f_Y(y), \quad \theta \in [-1, 1]$$

  with the marginal interclaim p.d.f. $f_V(t) = \lambda e^{-\lambda t}$ $(\lambda > 0)$ and $F_V(t) = \int_0^t f_V(s) ds$, and marginal claim size p.d.f. f_Y and $F_Y(y) = \int_0^y f_Y(x) dx$. It follows from [3] that the above bivariate p.d.f. can be written as

$$f_{V,Y}(t, y) = e^{-\lambda t} \lambda f_Y(y) + \sum_{j=1}^{c+1} e^{-\beta_j t} [\beta_j \alpha_j \theta h_1'(F_Y(y)) f_Y(y)], \tag{1.5}$$

  where $\beta_j = \lambda(d + j - 1)$, and $\alpha_j = c!\lambda^c(-1)/(\prod_{i=1, i\neq j}^{c+1}(\lambda_i - \lambda_j))$. Hence, it also takes the form of (1.2).

The study on dividend strategies for insurance risk models has a long history since [12]. One popular strategy is the threshold dividend strategy, where the insurance company pays off a percentage of premium income as dividends whenever the current surplus is larger than a given threshold. Lin and Pavlova [13] study the compound Poisson risk model with threshold dividend strategy. [14] considers the threshold dividend strategy in the compound Poisson risk model perturbed by a Brownian motion, which has been extended by Chi and Lin [15] to a generalized jump–diffusion risk model. For the Sparre Andersen risk model, more attention is paid to the case where the interclaim times follow (generalized) Erlang distribution; see e.g. [16–19]. The Erlang assumption makes the analysis of ruin functions more convenient. However, for some other risk models (such as the renewal risk model with mixture-of-exponentials interclaim times), the usual arguments (see e.g. [18]) cannot be used. In this paper, we will follow an alternative approach to treat this problem in a perturbed Sparre Andersen risk model with dependence and threshold dividend strategy.

Assume that dividends are paid off by a constant rate $0 < \alpha < c$ whenever the surplus is above a threshold $b > 0$. Under this strategy, the modified surplus process, denoted by U_t^b, is defined as

$$dU_t^b = \begin{cases} cdt - dS_t + \sigma dB_t, & 0 \le U_t^b \le b, \\ (c - \alpha)dt - dS_t + \sigma dB_t, & U_t^b > b, \end{cases} \tag{1.6}$$

where $S_t = \sum_{i=1}^{N_t} Y_i$ is the aggregate claims process. Note that when the threshold b goes to infinity, the above surplus process reduces to model (1.1).

The ruin time τ_b is defined to be the first time when the surplus process (1.6) becomes negative, i.e.

$$\tau_b = \inf\{t \ge 0 : U_t^b < 0\}.$$

Throughout this paper, we assume that the following net profit condition holds,

$$\mathbb{E}[(c - \alpha)V - Y] > 0.$$