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Higher-order symmetric duality under cone-invexity and other related concepts

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ABSTRACT

In this paper we establish weak, strong and converse duality results for a pair of Wolfe type higher-order symmetric dual problems over cones under the assumption of higher-order cone-invexity. We also introduce the concepts of higher-order strictly and strongly cone-pseudoinvexity and use them to obtain weak, strong and converse duality results for the pair of Mond–Weir type higher-order symmetric dual problems.

1. Introduction

A pair of primal and dual problems is said to be symmetric if the dual of dual is primal. First order symmetric duality for differentiable mathematical programs has been studied by many authors like Dantzig et al. [1], Bazaraa and Goode [2] and Chandra and Kumar [3]. Kim et al. [4] formulated a pair of Wolfe type multiobjective symmetric dual programs for pseudoinvex functions with cone constraints. Suneja et al. [5] studied a pair of symmetric dual multiobjective programs of Wolfe type over arbitrary cones in which the objective function has been optimized with respect to an arbitrary closed convex cone by assuming the functions involved to be cone-convex. Later Khurana [6] established duality results for a pair of Mond–Weir type symmetric dual programs over arbitrary cones under the assumptions of cone-invexity, cone-pseudoinvexity and strongly cone-pseudoinvexity.

The study of second and higher-order duality is significant due to the computational advantage over first order duality as it provides tighter bounds for the value of the objective function when approximations are used. Mangasarian [7] introduced second and higher-order duality for nonlinear programming problems. Bector and Chandra [8] established symmetric and self duality results for second-order primal and dual programs by assuming the functions to be pseudo-bonvex and pseudo-boncave. Then Gulati et al. [9] studied both the second-order Wolfe type and Mond–Weir type nonlinear symmetric dual programs under the assumptions of η-convexity and η-pseudoconvexity and later the same problems were considered by Gulati et al. [10] by taking the constraints to be in arbitrary closed cone of $\mathbb{R}^n$ instead of non-negative orthant. Suneja et al. [11] considered a pair of multiobjective second order symmetric dual problems of Mond–Weir type and established weak, strong and converse duality results under the assumptions of η-bonvexity and η-pseudo-bonvexity. Gulati et al. [12] studied second-order Wolfe type and Mond–Weir type multiobjective symmetric

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dual pair of problems with cone constraints by using η-bonvexity and η-pseudo-bonvexity. Ahmad and Husain [13] studied second-order symmetric duality in multiobjective programming involving cones. Gulati and Gupta [14] considered a pair of higher-order nonlinear symmetric dual programs and proved the duality results under the assumptions of higher-order η-invexity/η-pseudoinvexity. Padhan and Nahak [15] established various higher-order duality results (weak, strong and converse duality) for a pair of Wolfe type and Mond–Weir type higher-order multiobjective symmetric dual problems, under higher-order invexity and pseudoinvexity assumptions. Ahmad [16] formulated a unified higher order dual for a nondifferentiable multiobjective programming problem and presented duality results under suitable conditions. Agarwal et al. [17] formulated a pair of Mond–Weir type nondifferentiable multiobjective higher order symmetric dual programs over arbitrary cones and established weak, strong and converse duality theorems under higher order K-F-convexity assumptions. Recently, Gupta and Jayswal [18] studied the higher-order Mond–Weir type multiobjective symmetric duality over cones using higher-order cone-preinvex (called invex at some places) and cone-pseudoinvex functions.

In this paper we have formulated a pair of Wolfe type higher-order symmetric dual problems and established the weak, strong and converse duality results by assuming the concerned functions to be higher-order cone-invex. We have also introduced the notions of higher-order strictly and strongly cone-pseudoinvexity and used them to prove the weak duality result for the Mond–Weir type higher-order symmetric dual pair of problems given by Gupta and Jayswal [18]. They have established the strong duality result by considering only those cones which contain the non-negative orthant of $\mathbb{R}^k$ whereas we have proved this result for arbitrary cones in $\mathbb{R}^k$.

2. Notations and definitions

Let S_1 and S_2 be non-empty open sets in $\mathbb{R}^n$ and $\mathbb{R}^m$, respectively and C_1 and C_2 be closed convex cones with non-empty interiors in $\mathbb{R}^n$ and $\mathbb{R}^m$, respectively such that $C_1 \times C_2 \subseteq S_1 \times S_2$. Let K be a closed convex pointed cone in $\mathbb{R}^k$ such that $\text{int}K \neq \phi$, where $\text{int}K$ denotes the interior of K. The positive dual cone K^+ of K is defined as follows:

$$K^+ = \{y \in \mathbb{R}^m : x^T y \geq 0, \text{ for all } x \in K\}.$$

For each $i = 1, 2, \ldots, k$, let $f_i : S_1 \times S_2 \longrightarrow \mathbb{R}$, $h_i : S_1 \times S_2 \times \mathbb{R}^m \longrightarrow \mathbb{R}$ and $g_i : S_1 \times S_2 \times \mathbb{R}^n \longrightarrow \mathbb{R}$ be differentiable functions. Let $\eta_1 : S_1 \times S_1 \longrightarrow \mathbb{R}^n$ and $\eta_2 : S_2 \times S_2 \longrightarrow \mathbb{R}^m$ be any vector-valued functions.

We will be using the following notations in this paper

- $p = (p_1, p_2, \ldots, p_k)$ and $r = (r_1, r_2, \ldots, r_k)$, for each $p_i \in \mathbb{R}^m$ and $r_i \in \mathbb{R}^n$, $i = 1, 2, \ldots, k$
- $f(x, y) = (f_1(x, y), f_2(x, y), \ldots, f_k(x, y))$
- $\nabla_x f_i(\bar{x}, \bar{y})$ denotes the $n \times 1$ gradient vector with respect to x at $(\bar{x}, \bar{y})$, $\nabla_{xx} f_i(\bar{x}, \bar{y})$ and $\nabla_{xy} f_i(\bar{x}, \bar{y})$ denote the $n \times n$ and $m \times n$ matrices of second order partial derivatives at $(\bar{x}, \bar{y})$, respectively.
- $h(x, y, p) = (h_1(x, y, p_1), h_2(x, y, p_2), \ldots, h_k(x, y, p_k))$,
  $g(u, v, r) = (g_1(u, v, r_1), g_2(u, v, r_2), \ldots, g_k(u, v, r_k))$
- $\nabla_p h(x, y, p) = \big(\nabla_{p_1} h_1(x, y, p_1), \nabla_{p_2} h_2(x, y, p_2), \ldots, \nabla_{p_k} h_k(x, y, p_k)\big)$,
  $\nabla_r g(u, v, r) = \big(\nabla_{r_1} g_1(u, v, r_1), \nabla_{r_2} g_2(u, v, r_2), \ldots, \nabla_{r_k} g_k(u, v, r_k)\big)$
- $p^T \nabla_p h(x, y, p) = \big(p_1^T \nabla_{p_1} h_1(x, y, p_1), p_2^T \nabla_{p_2} h_2(x, y, p_2), \ldots, p_k^T \nabla_{p_k} h_k(x, y, p_k)\big)$ and
  $r^T \nabla_r g(u, v, r) = \big(r_1^T \nabla_{r_1} g_1(u, v, r_1), r_2^T \nabla_{r_2} g_2(u, v, r_2), \ldots, r_k^T \nabla_{r_k} g_k(u, v, r_k)\big)$.

The following definition is based on the Definition 2.3 of Gupta and Jayswal [18].

Definition 2.1. The function $f(x, \cdot)$ is said to be higher-order K-invex at $y \in S_2$ with respect to h and η_2 for a fixed x, if for every $w \in S_2$ and $p_i \in \mathbb{R}^m$, $i = 1, 2, \ldots, k$

$$f(x, w) - f(x, y) - h(x, y, p) + p^T \nabla_p h(x, y, p) - \eta_2(w, y)^T \left[\nabla_y f(x, y) + \nabla_p h(x, y, p)\right] \in K.$$

3. Wolfe type higher-order symmetric duality

We consider the following Wolfe type higher-order symmetric dual pair, where $e = (e_1, e_2, \ldots, e_k) \in \text{int}K$ is fixed.

(WP) $K\text{-Min}\, f(x, y) + h(x, y, p) - p^T \nabla_p h(x, y, p) - y^T \sum_{i=1}^k \lambda_i \{\nabla_y f_i(x, y) + \nabla_{p_i} h_i(x, y, p_i)\} e$
subject to

$$-\sum_{i=1}^{k} \lambda_i \{\nabla_y f_i(x, y) + \nabla_{p_i} h_i(x, y, p_i)\} \in C_2^+, \tag{1}$$

$$\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_k) \in \text{int}K^+, \lambda^T e = 1, x \in C_1, p_i \in \mathbb{R}^m, i = 1, 2, \ldots, k.$$