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A Riemannian subgradient algorithm for economic dispatch with valve-point effect☆

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A B S T R A C T

The economic load dispatch problem (ELDP) is a classical problem in the power systems community. It consists in the optimal scheduling of the output of power generating units to meet the required load demand subject to unit and system inequality and equality constraints. This optimization problem is challenging on three different levels: the geometry of its feasible set, the non-differentiability of its cost function and the multimodal aspect of its landscape. For this reason, ELDP has received much attention in the past few years and numerous derivative-free techniques have been proposed to tackle its multimodal and nondifferentiable characteristics. In this work we propose a different approach, exploiting the rich geometrical structure of the problem. We show that the (nonlinear) equality constraint can be handled in the framework of Riemannian manifolds and we develop a feasible (all iterates satisfy the constraints) subgradient descent algorithm to provide fast convergence to local minima. To this end, we show that Clarke's calculus can be used to compute a deterministic admissible descent direction by solving a simple, low-dimensional quadratic program. We test our approach on four real data sets. The proposed method provides fast local convergence and scales well with respect to the problem dimension. Finally, we show that the proposed algorithm, being a local optimization method, can be incorporated in existing heuristic techniques to provide a better exploration of the search space.

1. Introduction

The economic load dispatch problem (ELDP) is the optimal scheduling of the output of power generating units to meet the required load demand subject to unit and system equality and inequality constraints [1]. In traditional ELDP, the cost function for each generator is modeled by a single quadratic function. Nevertheless, in practice, one has to take into account highly nonlinear input–output characteristics arising due to valve-point loadings or generating unit ramp rate limits. As a consequence, we end up with a nonsmooth, equality- and inequality-constrained optimization problem, (5), which is in general multimodal (it presents several local optima) and for which classical smooth optimization techniques are thus not suitable.

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For this reason, ELDP has received much attention in the past few years and numerous derivative-free techniques have been proposed to tackle its multimodal and nondifferentiable aspects. Popular techniques include genetic algorithms (GA) [2], evolutionary programming [3], particle swarm optimization (PSO) [4], and differential evolution (DE) [5].

More recently, "global–local" hybrid methods have appeared that combine a "global" method having good global searching abilities and a "local" method with better fine-tuning abilities. Even though there may not be a clear frontier between the two groups of methods, it is quite evident that methods such as GA, PSO and DE belong to the "global" group whereas a method such as sequential quadratic programming (SQP) belongs to the "local" group. Desirable properties of a local method include the following. (i) The method should be *feasible*, i.e., all the iterates should satisfy the equality and inequality constraints. Indeed, a drawback of infeasible methods is that even if the outcome is close to satisfying the constraints, the cost function value at the outcome may be quite different from its value at the nearest point satisfying the constraints. Another downside of infeasibility is that the intermediate results are not directly usable since they do not respect the balance constraint. (ii) The iterates should converge to a local minimizer of the cost function. (iii) There is a well-known trade-off between low numerical cost of the iteration and fast convergence of the iterates; as a simplistic illustration, concatenating a few steps of a given method improves the speed of convergence per iterate but degrades the numerical cost per iterate. If the sought accuracy is sufficiently low – which is arguably the case of the ELDP where the coefficient values and even the model itself are debatable – then it is preferable to run a lot of cheaper iterations, as this makes it possible to check a stopping criterion more frequently and hence to avoid the unnecessary computational effort inherent to overconvergence. (iv) Methods that exploit the very particular structure of the ELDP should be preferred over "out of the box" numerical algorithms.

SQP is a popular choice for the local method in global–local ELDP algorithms; see, e.g., [6–9]. Whereas SQP is a welcome complement to global search methods, it does not satisfy any of the properties mentioned above. Specifically, it is an infeasible method, since the equality constraint is only satisfied in the limit. Convergence, while empirically observed, is not guaranteed because the classical convergence theory of SQP assumes, among other things, that the cost function and the constraint functions have continuous first derivatives [10, Theorem 18.3], which is not the case in the ELDP (5). As a quadratically convergent method [10, Theorem 18.4], it can be seen as favoring fast convergence of the iterates at the expense of a higher numerical cost per iterate. Finally, it is a general-purpose algorithm, not specifically tailored to the ELDP.

Other local methods have been proposed in combination with global methods. The Nelder–Mead (NM) method has been shown to be an effective local method in combination with PSO [11], but it does not exploit the readily accessible first-order information on the cost function. The modified subgradient (MSG) method used in [12] exploits first-order information on the cost function, but feasibility of the iterates is achieved only after a certain number of steps in view of the *sharp* augmented Lagrangian approach. Shor's r-algorithm, combined with an improved differential evolution (IDE) method in [13], is also of the subgradient type, but the iterates are not feasible.

In this paper, we develop a novel method that satisfies properties (i), (iii), and (iv). It is also built to satisfy (ii) and does it empirically, however, as a consequence of (iv), its convergence does not immediately follow from an existing result, and a detailed convergence analysis is beyond the scope of this paper.

Property (i), feasibility of the iterates, is enforced using the framework of Riemannian optimization. The adequacy of this framework stems from the fact that the equality constraint (3) defines an ellipsoid, which admits a natural structure of a Riemannian manifold. Riemannian optimization, also called optimization on manifolds, is a vibrant area of research nowadays, whose foundations can be found, e.g., in [14–16]. Since this framework is new in the ELDP context, we dedicate a significant part of this paper to laying out the necessary background.

Property (iii) comes by preferring a steepest-descent approach over a second-order approach. However, in view of the non-smoothness due to the valve-point effect, the steepest-descent approach does not rely on gradient techniques but rather on subgradient techniques, using the framework of Clarke's generalized calculus [17].

In view of the above, the proposed method fits in the framework of Riemannian subgradient descent. With respect to the general-purpose Riemannian subgradient descent method of Dirr et al. [18], a contribution of our development is to incorporate bound constraints in order to handle the generator capacity constraints (2) present in the ELDP. Another contribution of this work is that, whereas many heuristic algorithms for the ELDP consist of (a combination of) existing black-box optimization techniques, the proposed method strives to exploit as much as possible the very particular structure of the ELDP. This allows notably for an efficient representation of the generalized gradient, which enables fast computation of a descent direction by solving a low-dimensional quadratic program.

In summary, the proposed technique provides fast convergence to a nearby local minimum of the ELDP (5), while satisfying the power balance (3) and capacity constraints (2) throughout the optimization process. Therefore, the aforementioned heuristics largely explored in the literature and the proposed subgradient descent algorithm present very complementary properties: the multimodal aspect of the ELDP can be addressed using any global feasible exploration tool while the local refinement of a potential solution is efficiently provided by the proposed approach, including a check for the stationarity of the final iterate.

The remainder of the paper is organized as follows. In Section 2, the ELDP with the valve-point effect is briefly presented, followed by a detailed treatment of the underlying geometry of the optimization problem and an introduction to the necessary differential geometry tools in Section 3. The subgradient descent algorithm is presented in Section 4. Subsequently, its formulation on the Riemannian manifold and its specialization for the ELDP are discussed. Section 5 presents the