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On second-order duality for nondifferentiable minimax fractional programming

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ABSTRACT

Two types of second-order dual models for a nondifferentiable minimax fractional programming problem are formulated and proved weak, strong, strict converse duality theorems using η-bonvexity assumptions. Special cases are also discussed to show that this work extends some known results of the literature.

1. Introduction

Optimization problems, in which both a minimization and a maximization process are performed, are known as minimax or minmax problems in the area of mathematical programming. These problems are used in decision theory, game theory, statistics and philosophy for minimizing the possible loss for a worst case (maximum loss) scenario. For the theory, algorithms and applications of some minimax problems, the reader is referred to [1]. Schmitendorf [2], established necessary and sufficient optimality conditions for such problems. Tanimoto [3] used these optimality conditions to construct a dual problem and proved duality relations involving convex functions. Bector and Bhatia [4] and Weir [5] relaxed the study in [2] to pseudo/quasi-convexity. In the last two decades, several authors have shown their interest in developing optimality conditions and various duality results for minimax fractional programming problems dealing with the differentiable case in [6–12] and the nondifferentiable case in [13–19].

For a nondifferentiable minimax fractional programming problem, Ahmad and Husain [17] proved duality results under (F, α, ρ, d)-pseudoconvex functions, Jayswal [19] discussed different dual models under generalized α-univexity and Lai and Chen [20] studied Mond–Weir and Wolfe type dual models.

On second-order minimax programming problems, several researchers [9–11,21,22] have worked. Husain et al. [9] established duality relations for two types of second-order dual models of a minimax fractional programming problem involving η-bonvex functions. These models were later on generalized in [10] by introducing an additional vector r. Recently, Sharma and Gulati [11] introduced second-order α-type I univexity and further studied duality results for the dual problems considered in [9].

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In this paper, we first formulate two types of second-order dual models for a nondifferentiable fractional minimax problem and then establish weak, strong and strict converse duality theorems under η-bonvexity assumptions. This paper has been divided into four sections. Section 2 includes the nondifferentiable minimax fractional problem and some notations and preliminaries. In the next two sections, we consider two second-order dual models for the problem discussed in Section 2 and obtained usual duality results involving η-bonvex functions.

2. Notations and preliminaries

Consider the following nondifferentiable minimax fractional programming problem:

$$\text{(P) Minimize } \psi(x) = \sup_{y\in Y} \frac{f(x,y) + (x^T Bx)^{1/2}}{h(x,y) - (x^T Dx)^{1/2}}$$

subject to $g(x) \leq 0$,

where Y is a compact subset of R^l, $f : R^n \times R^l \to R$, $h : R^n \times R^l \to R$ are twice continuously differentiable on $R^n \times R^l$ and $g : R^n \to R^m$ is twice continuously differentiable on R^n, B and D are $n\times n$ positive semidefinite matrix, $f(x,y)+(x^T Bx)^{1/2} \geq 0$ and $h(x,y) - (x^T Dx)^{1/2} > 0$ for each $(x,y) \in \mathfrak{J} \times Y$, where $\mathfrak{J} = \{x \in R^n : g(x) \leq 0\}$.

For each $(x,y) \in \mathfrak{J} \times Y$, we define

$$J(x) = \{j \in M = \{1, 2, \ldots, m\} : g_j(x) = 0\},$$

$$Y(x) = \left\{ y \in Y : \frac{f(x,y) + (x^T Bx)^{1/2}}{h(x,y) - (x^T Dx)^{1/2}} = \sup_{z\in Y} \frac{f(x,z) + (x^T Bx)^{1/2}}{h(x,z) - (x^T Dx)^{1/2}} \right\},$$

$$K(x) = \left\{ (s,t,\widetilde{y}) \in N \times R^s_+ \times R^{ls} : 1 \leq s \leq n+1,\ t = (t_1, t_2, \ldots, t_s) \in R^s_+, \right.$$
$$\left. \sum_{i=1}^{s} t_i = 1,\ \widetilde{y} = (\widetilde{y}_1, \widetilde{y}_2, \ldots, \widetilde{y}_s),\ \widetilde{y}_i \in Y(x),\ i = 1, 2, \ldots, s \right\}.$$

Let $\psi : X \to R$ ($X \subseteq R^n$) be a twice differentiable function.

Definition 2.1 (*[10]*). The function ψ is said to be η-bonvex at $u \in R^n$, if there exists $\eta : X \times X \to R^n$ such that for all $x, p \in R^n$, we have

$$\psi(x) - \psi(u) + \frac{1}{2} p^T \nabla^2 \psi(u) p \geq \eta^T(x,u)[\nabla\psi(u) + \nabla^2\psi(u)p].$$

Definition 2.2. The function ψ is said to be strictly η-bonvex at $u \in R^n$, if there exists $\eta : X \times X \to R^n$ such that for all $x, p \in R^n$ and $x \neq u$, we have

$$\psi(x) - \psi(u) + \frac{1}{2} p^T \nabla^2 \psi(u) p > \eta^T(x,u)[\nabla\psi(u) + \nabla^2\psi(u)p].$$

Lemma 2.1 (*Generalized Schwarz Inequality*). *Let B be a positive semidefinite matrix of order n. Then, for all $x,\ w \in R^n$,*

$$x^T Bw \leq (x^T Bx)^{1/2} (w^T Bw)^{1/2}.$$

The equality holds if $Bx = \lambda Bw$ for some $\lambda \geq 0$.

The following Theorem 2.1 [13, Theorem 3.1] will be required to prove strong duality theorems:

Theorem 2.1 (*Necessary Condition*). *If x^* is an optimal solution of problem* (P) *satisfying $x^{*T} Bx^* > 0, x^{*T} Dx^* > 0$, and $\{\nabla g_j(x^*), j \in J(x^*)\}$ are linearly independent, then there exist $(s^*, t^*, \widetilde{y}^*) \in K(x^*), k_0 \in R_+,\ w,\ v \in R^n$ and $\mu^* \in R^m_+$ such that*

$$\sum_{i=1}^{s^*} t_i^* \{\nabla f(x^*, \widetilde{y}_i^*) + Bw - k_0(\nabla h(x^*, \widetilde{y}_i^*) - Dv)\} + \sum_{j=1}^{m} \mu_j^* \nabla g_j(x^*) = 0, \tag{1}$$

$$f(x^*, \widetilde{y}_i^*) + (x^{*T} Bx^*)^{1/2} - k_0(h(x^*, \widetilde{y}_i^*) - (x^{*T} Dx^*)^{1/2}) = 0, \quad i = 1, 2, \ldots, s^*, \tag{2}$$

$$\sum_{j=1}^{m} \mu_j^* g_j(x^*) = 0, \tag{3}$$