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Linear bilevel programming with interval coefficients☆

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ABSTRACT

In this paper, we address linear bilevel programs when the coefficients of both objective functions are interval numbers. The focus is on the optimal value range problem which consists of computing the best and worst optimal objective function values and determining the settings of the interval coefficients which provide these values. We prove by examples that, in general, there is no precise way of systematizing the specific values of the interval coefficients that can be used to compute the best and worst possible optimal solutions. Taking into account the properties of linear bilevel problems, we prove that these two optimal solutions occur at extreme points of the polyhedron defined by the common constraints. Moreover, we develop two algorithms based on ranking extreme points that allow us to compute them as well as determining settings of the interval coefficients which provide the optimal value range.

1. Introduction

Bilevel programming involves two optimization problems where the constraint region of one of the problems is implicitly determined by the other. Bilevel problems have been proposed for dealing with hierarchical processes involving two levels of decision making and have been increasingly addressed in the literature. They can be formulated as follows:

$$\begin{aligned} \min_{x,y} \quad & f_1(x, y), \quad \text{where } y \text{ solves} \\ \min_{y} \quad & f_2(x, y) \\ \text{s.t.} \quad & (x, y) \in S \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^{n_1}$ are the upper level variables, which are controlled by the upper level decision maker; $y \in \mathbb{R}^{n_2}$ are the lower level variables, which are controlled by the lower level decision maker; $f_1, f_2 : \mathbb{R}^{n_1+n_2} \longrightarrow \mathbb{R}$ are the upper level and lower level objective functions, respectively; $S \subseteq \mathbb{R}^{n_1+n_2}$ is the common constraint region. Due to their structure, bilevel programs are nonconvex and quite difficult to deal with and solve. In fact, even the simplest model in bilevel programming, the linear bilevel program, in which all functions involved are linear, is strongly NP-hard [1]. Colson et al. [2], Dempe [3] and Vicente and Calamai [4] provide surveys on the subject. Bard [5], Dempe [6] and Shimizu et al. [7] are good textbooks on this topic.

In the above mathematical formulation of the problem, the coefficients are assumed to be known exactly. However, in practice, it is very common for the coefficient values to be only approximately known. When there is one single level of decision making, several approaches have been proposed in the literature to describe and treat imprecise and uncertain elements present in decision problems. Fuzzy programming and stochastic programming are frequently used to tackle the

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problem of inexactness in coefficients. The former assumes that membership functions of fuzzy parameters are known. The latter requires probability distributions of coefficients to be known. However, in some real-world cases it might be difficult for decision makers to specify any of these assumptions.

To overcome these difficulties, mathematical programs whose parameters are only assumed to be in intervals have been considered in the literature, almost exclusively in linear programming. Some references consider only interval numbers in the objective function coefficients of a linear problem. Among them, Steuer [8] proposes algorithms for determining all extreme points and unbounded edge directions that solve the problem for at least one setting of the coefficient values in their ranges. Taking a different approach, Ishibuchi and Tanaka [9] assume that decision makers have preferences when selecting intervals. They define several partial order relations which represent the decision maker's preferences between intervals and propose their use for selecting optimal solutions in minimum linear programs with interval coefficients in the objective function. Finally, Inuiguchi and Sakawa [10] and Mausser and Laguna [11] propose to find a solution that will be close to optimal, regardless of the values eventually taken by the coefficients of the objective function. Hence, they approach the problem by using the minimax or the minimax relative regret criterion.

Mathematical programs with interval coefficients in the objective function as well as in the constraints have also been addressed in the literature. In this case, the focus is on the problem of computing the optimal value range, i.e., the range between the best and the worst optimal objective function values and the settings of the coefficients which provide the two extreme cases. These two extreme values allow the decision maker to better understand the risk involved and to gain an insight into the likelihood of these extremes [12]. For linear problems, Shaocheng [13] determines the linear programs which provide the best and worst possible optimal solutions when all variables are nonnegative variables and all constraints are inequalities. For the same problem, Chinneck and Ramadan [12] extend this study by comprehensively analyzing all kinds of variables and constraints and providing algorithms which allow them to determine the best and worst optimum and the coefficients which achieve these two extremes. Based on the properties of linear systems with interval coefficients, Hladík [14] proposes a unified approach for the problem of computing the optimal value range in linear problems with interval coefficients that allows for some dependences between coefficients. Very few results have been obtained for nonlinear problems. In fact, only nonlinearities in the objective function have been analyzed in [15] in linear fractional programs and in [16] in convex quadratic problems. Notice that mathematical programming with interval coefficients can be considered as an extension of classical sensitivity analysis as it deals with simultaneous and independent perturbations of the parameters.

This paper addresses linear bilevel problems whose objective function coefficients are assumed to lie between specific bounds. That is to say, f_1 and f_2 are linear functions with interval coefficients and the common constraint region S is a polyhedron. The purpose of this paper is to solve the optimal value range problem. To our knowledge, this is the first time that interval coefficients in bilevel programming have been considered. Notice that although the term linear is used in the description of these bilevel problems, the fact that the feasible region is implicitly determined by another mathematical program makes them nonlinear problems. This characteristic prevents the use of the properties of linear systems which form the base of the techniques used in previous works in mathematical programming with interval coefficients and makes the study of linear bilevel problems with interval coefficients more difficult. Focusing on the optimal value range problem, we will prove that the best and worst optimal solutions with respect to the upper level objective function occur at extreme points of the polyhedron S. Moreover, we will develop two enumerative algorithms that allow us to compute them as well as determining settings of the interval coefficients which provide these values.

The paper is organized as follows. Section 2 states the linear bilevel problem with interval coefficients (LBPIC). Section 3 goes on to analyze the LBPIC when the interval coefficients are only in the upper level objective function. Taking into account the properties of linear bilevel problems, the optimal value range can be obtained by solving two linear bilevel problems. In Section 4, the LBPIC is analyzed when the interval coefficients are only in the lower level objective function. In this case, the feasible region depends on the intervals, thus making the study more complex. Two algorithms are proposed to obtain the best and worst optimal solutions based loosely on the ideas of ranking extreme points of the Kth-best algorithm. In Section 5, the approaches developed when analyzing previous cases separately are integrated to obtain the optimal value range for the LBPIC. Finally, our conclusions are presented in Section 6.

2. LBPIC problem formulation

Linear bilevel programming has been studied very extensively in the literature (see [17,18,3] and the references therein). The linear bilevel problem can be formulated as:

$$\min_{x,y} \ cx + dy \quad \text{where } y \text{ solves} \tag{2a}$$

$$\min_{y} \ ay \tag{2b}$$

$$\text{s.t. } Ax + By \leqslant b \tag{2c}$$

$$x \geqslant 0, y \geqslant 0, \tag{2d}$$